## Competition Among Heterogenous Firms

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## Chapter 1

## Introduction

This thesis covers competition among heterogenous firms. The majority of papers in industrial organization build on models characterized by homogenous firms. The main reason is twofold. Firstly, specific effects can easier be isolated in a simplified model like one with homogenous firms. Secondly, the computational complexity significantly increases when introducing heterogenous firms and therefore tackling specific problems gets more difficult. However, firms are different, i.e. heterogenous, in real world markets. A model with homogenous firms builds on the unrealistic assumption that firms are duplications of each other. For that reason, this thesis accounts for heterogeneity of firms and therefore allows to present models that can much better derive statements regarding real world behavior and its implications.

This thesis has three main chapters that each considers different questions in industrial organization regarding competition among heterogenous firms.

Chapter 2 entitled Competition between Pay-TV and Public Service Broadcasting:

A Two-Sided Market Analysis investigates the behavior of two competing TV channels Pay-TV and public service broadcasting (PSB) that coexist in most EU countries. While PayTV channels have to finance themselves by advertising and subscription revenue generated from viewers, public service broadcasters are financed by advertising income and public funds.
TV channels both bring together advertisers and viewers in a situation in which advertisers are interested in many viewers watching their adverts but viewers dislike advertising and may switch to the competing channel. The economic literature has not explored the consequences caused by the coexistence of a Pay-TV channel and a public service broadcaster (PSB) in a setting where

both channels receive payments from viewers directly so far. Additionally, we assume that consumers engage in mental accounting and propose a model applying a portfolio approach, which, until now has not yet been used for modeling television markets. The model analyzes how the level of the broadcasting fee and the 'nuisance factor' associated with advertising from the viewer's point of view affect the behavior of the channels and market outcomes. It turns out that the Pay-TV channel decides to show no adverts altogether if viewers display a strong aversion to advertising. If the broadcasting fee is sufficiently high, the Pay-TV channel switches to a free-to-air channel and the PSB does not show any adverts at all.

Chapter 3 entitled Endogenous Merger Formation and Incentives to Invest in Cost Reducing Innovations analyzes how process innovations affect merger incentives, using an endogenous horizontal merger model of three firms. Firms are heterogenous in their process innovation technologies, i.e. they differ in the cost of conducting R&D that leads to a reduction in production cost. The theoretical literature does not analyze the relationship between mergers and innovation where the merger decision is endogenized. We apply a cooperative bargaining approach to analyze the merger pattern. We find that the two most efficient firms merge if innovation is not too expensive, while the least efficient firm remains independent. For high innovation cost levels no merger will take place. Accounting for endogenous merger formation R&D subsidies are not necessarily positive depending on the R&D efficiency in the industry.

Chapter 4 entitled **Horizontal Divestitures and R&D Incentives in Asymmetric Duopoly** analyzes how a threat of horizontal divestiture affects R&D incentives and welfare in an asymmetric Cournot duopoly where an efficient low-cost firm competes against a less efficient high-cost firm. Firms account for a possible divestiture of the low-cost firm. In our 2-stage model, both firms can not only choose output, but also decrease their marginal cost via process innovations (stage 1). Without innovation, firms face different (constant) marginal cost of production. Between stages 1 and 2, there might be a divestiture of the low-cost firm. In case of divestiture firms face a third low-cost competitor and again compete in Cournot fashion. While there is a broad literature on the relationship between market structure and R&D incentives, there is no literature that endogenizes R&D investments and potential divestitures. We find that an actual divestiture measure harms both the high- and low-cost firm and it only improves welfare if the low-cost firm sufficiently dominates the high-cost firm. The industry-wide rate of

innovation is lowered if a divestiture becomes more likely.

## Chapter 2

# Competition between Pay-TV and Public Service Broadcasting: A Two-Sided Market Analysis

#### 2.1 Introduction

In most EU countries commercial TV channels and public service broadcasters coexist. Pay-TV channels have two sources of revenue, advertising and subscription revenue. Public service broadcasters are financed by advertising income and public funds. However, Pay-TV channels and public service broadcasters compete in the same market. Over the last few years, there has been an intense discussion as to whether public funding of broadcasting in Europe harms commercial rivals and should therefore be limited.

Today, television markets are analyzed within the theory of two-sided markets and intermarket network effects. Advertisers and viewers are brought together in a situation in which advertisers are interested in many viewers watching their adverts but viewers dislike advertising. Most of the literature on TV broadcasting markets, within the framework of two-sided markets, only focuses on competition between two commercial stations financed by advertising and the rare articles incorporating public service broadcasting (PSB) do not account for Pay-TV. Stühmeier and Wenzel (2010) analyze a setting in which a commercial channel competes with a public service broadcaster (PSB) but do not allow for any direct payments from viewers while

Kohlschein (2005) accounts for broadcasting fees but the commercial channel also cannot charge viewers directly. Accordingly, the economic literature has not explored the consequences caused by the coexistence of a Pay-TV channel and a PSB in a setting where both channels receive payments from viewers directly so far. Additionally, a standard approach applying a Hotelling model has emerged as a canonical setup in the literature. As a contrast, we assume that consumers engage in mental accounting and propose a model applying a portfolio approach, which, until now has not yet been used for modeling television markets. Mental accounting (see Thaler, 1980, 1985) follows the following logic: Prior to developing a demand for a particular good or service, consumers decide on a certain amount of money to be spent on a certain activity like watching TV. In our model we present the behavior of two competing channels - Pay-TV and PSB - applying a portfolio approach in order to model viewing demand. The model analyzes how the level of the broadcasting fee and the 'nuisance factor' associated with advertising from the viewer's point of view affect the behavior of the channels and market outcomes. The issue of whether a PSB favors a high public broadcasting fee to squeeze the Pay-TV channel out of the market is also analyzed.

This paper adds to recent literature on TV broadcasting concerning the theory of two-sided markets. Within the wider literature on two-sided markets different settings have recently been synthesized by Rochet and Tirole (2006) and Armstrong (2006). Television broadcasting within the framework of two-sided markets has been modeled in several papers in which broadcasters compete in program content and advertising levels. Gal-Or and Dukes (2003), Gabszewicz et al. (2004), Anderson and Coate (2005) and Peitz and Valletti (2008) offer similar theoretical settings. Viewers dislike advertising; viewer demand is modeled by applying the canonical Hotelling setup; broadcasters compete for viewers in a symmetric duopoly setting; platforms sell advertising quantities in their channels as monopolists; and variable costs are zero per assumption.

Gal-Or and Dukes (2003) analyze the incentives of two commercial media stations to reduce the extent of programming differentiation. As media stations lower differentiation between their programs, firms decrease their advertising expenses. Less product information is provided to viewers then. They are less informed about the options in the market. Hence, competition in the product market is reduced and producers gain higher profit margins. Accordingly, media stations can ask for higher payments from advertising.

Gabszewicz et al. (2004) offer a model with homogenous advertisers where two TV channels maximally differentiate in programming and viewers can watch a combination of two programs. However, as the disutility from advertising exceeds some critical value, TV channels no longer maximally differentiate. Programming of the two channels should be differentiated totally from a social point of view in order to give viewers the chance to select their ideal program combination.

Anderson and Coate (2005) develop a model with two commercial stations with exogenous content. Advertising is undersupplied from a social point if the nuisance caused by advertising is relatively low and advertising is oversupplied if the nuisance caused by advertising is relatively high.

Peitz and Valletti (2008) build on the model of Anderson and Coate (2005). In contrast, they allow for endogenous content choice. They compare the market outcomes of a duopoly setting with two commercial free-to-air channels financed by advertising only, and a duopoly setting with two competing Pay-TV channels financed by advertising and viewer subscription fees. If viewers dislike advertising, the advertising intensity is reduced with Pay-TV. The content is always maximally differentiated with Pay-TV while it is less differentiated with free-to-air. Equilibria in both markets do not maximize welfare. We have based our own model on their approach to modeling advertising.

Reisinger et al. (2009) use a model with differentiated free-to-air channels financed by revenues from advertising only. In contrast to the papers discussed so far, they account for both participation externalities, i.e. more advertising on one channel decreases (increases) its own (the other channels') viewer size, and pecuniary externalities, i.e. increased advertising level on one channel changes the advertisers' willingness to pay on all channels. They show that advertising can constitute a strategic substitute or complement and they present cases in which market entry increases the advertising level.

With respect to our model setup, Kohlschein (2005) and Rasch (2007) are the only papers using a formal two-sided market approach analyzing a setting with a PSB that receives public funds.

Kohlschein (2005) compares a symmetric duopoly setting for two competing commercial

channels financed by advertising only with an asymmetric duopoly setting for a commercial channel and a PSB. Content is given exogenously. Kohlschein shows that the introduction of public funding has two effects on viewers: on the negative side, they have to pay a broadcasting fee. On the positive side, they face decreasing advertising levels. Advertisers suffer as decreasing advertising levels increase ad prices. Their rent is reduced. The commercial station also suffers due to less viewers and advertising revenues. In contrast, the PSB benefits from viewers attracted away from the rival's station and a new source of income – the broadcasting fee. The overall welfare effect is ambiguous.

Rasch (2007) also compares a symmetric duopoly setting with two competing commercial channels with an asymmetric duopoly setting for a commercial channel and a PSB. In his work, Rasch is mainly interested in how the type of a commercial program selected from a socially preferable and a socially less preferable type of program changes when public funding is introduced. As one public broadcaster is introduced, which receives part of a broadcasting fee and has an obligation to show the socially preferable type of program, the commercial broadcaster may have incentives to opt for this type of program as well.

This work proceeds as follows: Section 2.2 presents the model in a simple version and equilibrium is derived. In section 2.3 the simple model is extended to a two-sided market model. Market exit of Pay-TV and consequences are the focus of section 2.4. In section 2.5, possible extensions are discussed before concluding with section 2.6.

#### 2.2 The Simple Model

In this section a simple model is introduced. Our presentation of the setup is built on an asymmetric duopoly model of a Pay-TV channel and a PSB. Channels compete for viewers. They are financed by two sources; advertising and direct payments from viewers. However, in contrast to the Pay-TV channel, the PSB cannot set the fee, i.e. the broadcasting fee, for its programming. Consumers engage in mental accounting and develop a portfolio demand. The simple model assumes viewers do not experience dissatisfaction when programs are interrupted in order to air commercials, and therefore cannot be considered a part of the two-sided market theory. Based on this simple model, an extended model, accounting for viewers who dislike

<sup>&</sup>lt;sup>1</sup>See section 2.1 and Thaler (1980, 1985).

advertising, is developed in section 2.3, so that markets are interrelated and two-sided.

#### 2.2.1 Viewers

The standard approach for analyzing the behavior of consumers is based on the assumption that they maximize utility, i.e. perfect rationality of consumers is assumed. The canonical setup applied in the literature on TV broadcasting presented in section 2.1, is based on this assumption as well. In contrast, we account for mental accounting assuming that a potential viewer sets a maximum individual monthly budget allocated to watching TV. Applying this portfolio approach, there may be cases where a viewer's decision does not maximize his utility, i.e. limited rationality. This approach has been described in the 'behavioral economics' literature by Ellison (2006) as rule-of-thumb approach.<sup>2</sup> According to Ellison, this approach is characterized by the assumption that consumer behavior is not motivated by finding a solution to a maximization problem but rather based on more straight forward decisions.

In our model, each viewer has an individual budget to spend on TV which is defined by his gross utility from watching TV. Let  $\mu$  be a viewer's budget to spend on TV which is distributed on the interval [0,1] according to a cumulative distribution function F which is uniformly distributed and continuously differentiable. The TV budget of a viewer is normalized to the interval [0,1] for computational simplicity without loss of generality. Thus, a viewer with a budget  $\mu=0$  cannot be considered as someone unwilling to pay anything for watching TV. Rather, his individual budget in monetary terms could be obtained using a specific formula, for example:  $(\mu+c) \cdot l$ , where c is a constant and l a constant factor. The variables denoting the fees for PSB, Pay-TV and advertising quantities are defined on the interval [0,1], as well. Thus, with Pay-TV, we do not allow for viewer subsidies, i.e. the subscription fee for Pay-TV cannot take negative values. This differs from the model by Peitz and Valletti (2008), who allow for viewer subsidies when analyzing competition between two Pay-TV channels.

In contrast to most papers, viewer demand is not set according to a Hotelling-style model as viewers do not decide between two channels. They face, instead, three options:

- 1. No subscription.
- 2. Subscription to PSB (mandatory broadcasting fee).

<sup>&</sup>lt;sup>2</sup>See Schmalensee (1978) and Smallwood and Conlisk (1979) for good examples of models applying the rule-of-thumb approach in industrial organization.

#### 3. Subscription to PSB and Pay-TV.

In line with a Hotelling-style model, viewers single-home, i.e. viewers who only subscribe to PSB only watch PSB and viewers who subscribe to both, PSB and Pay-TV, only watch Pay-TV. In contrast to a Hotelling-style model, viewers have to pay the broadcasting fee regardless of whether they subscribe to PSB only or to both PSB and Pay-TV. Pay-TV subscribers only watch Pay-TV in this model, although they are required to pay the broadcasting fee. Additionally viewers face a third option - subscribing to neither PSB nor Pay-TV.

One group of viewers with a low budget  $\mu$ , i.e. low gross utility from watching TV, only watches PSB, and one group of viewers with a high  $\mu$  watches Pay-TV. Additionally, there is a third group that does not subscribe to anything and therefore do not watch at all. The potential viewer side consists of mass N. Note that we disregard the influence of advertising levels on viewer utility in this simple model leaving this for a model extension to be discussed later. We also do not take differentiated contents into account. However, as viewers are assumed only to watch Pay-TV if they have subscribed to both, an implicit preference for the Pay-TV program schedule is assumed. It could be argued that a Pay-TV channel has to provide better programming in order to attract viewers to watch its programs. Let f be the broadcasting fee (e.g., "GEZ-Gebühr" in Germany) and s be the subscription fee for the Pay-TV channel. f and s can be set between 0 and 1, i.e. f and  $s \in [0,1]$ . An individual consumer makes his viewing decision according to the following allocation rule:

$$d_{PSB}(\mu) = \begin{cases} 0 & \text{if } \mu \ge f + s \lor \mu < f \\ 1 & \text{if } f \le \mu \le f + s \end{cases}$$
 (2.1)

$$d_{Pay-TV}(\mu) = \begin{cases} 0 & \text{if } \mu \le f+s \\ 1 & \text{if } \mu \ge f+s \end{cases}$$
 (2.2)

Hence, for the marginal subscriber to Pay-TV  $\hat{\mu}_{Pay-TV}$  we offer  $\hat{\mu}_{Pay-TV} = f + s$  and for the marginal subscriber to PSB we offer  $\hat{\mu}_{PSB} = f$  where  $\hat{\mu}_{Pay-TV}$  and  $\hat{\mu}_{PSB} \in [0,1]$ .

Let  $\delta$  denote the proportion of potential viewers that subscribe to both, PSB and Pay-TV,

thus  $0 \le \delta \le 1$ . As  $\mu$  is uniformly distributed on [0,1], we offer:

$$\delta = 1 - \hat{\mu}_{Pay-TV} = 1 - f - s \tag{2.3}$$

Let  $\phi$  denote the proportion of potential viewers that only subscribe to PSB. We get:

$$\phi = 1 - \delta - \hat{\mu}_{PSB} = 1 - \delta - f = s \tag{2.4}$$

The proportion of potential viewers that do not subscribe to anything is  $1 - \delta - \phi = f$ . Hence, the mandatory broadcasting fee determines the proportion of potential viewers that exit the market. As every TV owner has to pay a broadcasting fee f, the number of subscribers to PSB is independent from f, as long as  $s + f \le 1$ .

Accordingly, the viewing demand functions for the channels are

$$V_{Pay-TV} = N\delta \tag{2.5}$$

and

$$V_{PSB} = N\phi \tag{2.6}$$

where  $V_{Pay-TV}$  and  $V_{PSB}$  denote the number of viewers for Pay-TV and PSB respectively.

#### 2.2.2 Advertisers

The demand for advertising is based on Peitz and Valletti (2008). Advertisers sell products to viewers, who consume their products. All products are produced at constant marginal cost which is set equal to zero for simplicity. Consumer willingness to pay is equal to the quality  $\alpha$  of a product. Each advertiser is assumed to be a monopolist within his product market, and can, thus, extract full consumer surplus. In contrast to a simple monopoly model with decreasing demand, which calls for first order price differentiation to extract full consumer surplus, they assume homogenous viewers with respect to the willingness to pay for a specific product,  $\alpha$ . Producers differ with respect to the quality  $\alpha$ . Quality is distributed on an interval [0,1] according to a cumulative distribution function F with F(0) = 0 and F(1) = 1 which is

uniform on [0,1], and characterized by a continuously differentiable density.<sup>3</sup> Only consumers who have seen an advert buy a product. Advertisers multi-home, i.e. they can place adverts on none, one or both channels. Channel i provides advertising quantity  $a_i$  at a price  $r_i$  which is set by the model endogenously.

In contrast to Peitz and Valletti (2008), viewer demand is not set according to a Hotellingstyle model.  $\delta \in [0,1]$  is the proportion of viewers that watch Pay-TV and  $\phi \in [0,1]$  the proportion of viewers that watch PSB (see section 2.2.1). Thus, the profit for an advertiser of type  $\alpha$  from advertising on the Pay-TV channel is:

$$N\alpha\delta - r_1.$$
 (2.7)

The profit for an advertiser  $\alpha$  from advertising on the PSB channel is:

$$N\alpha\phi - r_2. (2.8)$$

As  $N\delta$  and  $N\phi$  are defined as the number of viewers that watch Pay-TV and PSB respectively, a viewer buys one product as a direct consequence of viewing the advert on TV. Advertising is profitable as long as a company gains additional profits due to the placement of an advert on a channel. For additional profits,  $N\alpha\delta > r_1$  and  $N\alpha\phi > r_2$  must be true for Pay-TV and PSB respectively. For the marginal advertiser  $\underline{\alpha}_1 = r_1/(N\delta)$  and  $\underline{\alpha}_2 = r_2/(N\phi)$ , we get  $N\alpha\delta = r_1$  for Pay-TV and  $N\alpha\phi = r_2$  for PSB. An advertiser of type  $\alpha < \underline{\alpha}_i$  will not place an advert on channel i as it is not profitable. The advertising quantity on the Pay-TV channel  $a_1$  and PSB channel  $a_2$  are:

$$a_1 = 1 - F(\underline{\alpha}_1) = 1 - r_1/(N\delta)$$
 (2.9)

and

$$a_2 = 1 - F(\underline{\alpha}_2) = 1 - r_2/(N\phi)$$
 (2.10)

having assumed the special case of an uniform distribution on the interval [0,1]. The amount of advertising  $a_1$  and  $a_2$  determine the advertising charge per viewer  $r_1/(N\delta)$  and  $r_2/(N\phi)$  for

<sup>&</sup>lt;sup>3</sup>Peitz and Valletti (2008) also consider the general case of  $\alpha$  being distributed on an interval  $[0, \alpha^{\max}]$ .

Pay-TV and PSB respectively. The decision regarding advertising quantity on one channel is not affected by the other channel's decisions.

#### 2.2.3 TV Channels

#### **PSB**

The profit function of the PSB in our model is closely related to Kohlschein (2005), who analyzed competition between a PSB and Free-TV channel in an asymmetric duopoly setting. The PSB is financed by two sources in Kohlschein's model: public funds and advertising income. The broadcasting fee, f, which is levied on all viewers regardless of whether they actually watch PSB, is collected by an independent institution, e.g., the "Gebührenzentrale (GEZ)" in Germany. In line with Kohlschein, the PSB is not able to influence the level of the broadcasting fee f in this simple model. This restriction will be relaxed in a model extension. In Kohlschein's model the broadcasting fee, f, is partially forwarded to the PSB depending on its success in attracting a large number of viewers. He argues that public service broadcasters with a small number of viewers are likely to be closed while those with a large number of viewers generate political goodwill and therefore get a larger proportion of the revenues from total broadcasting fees  $(V_{Pay-TV} + V_{PSB}) \cdot f = N(1-f) \cdot f$ . Hence, the amount of public funds received by the PSB depends on two factors, the size of the broadcasting fee, f, and the number of viewers watching PSB  $V_{PSB}$ . For the amount of public transfers, we have  $V_{PSB} \cdot f$ . Whilst similar to Kohlschein (2005), we provide a more sophisticated model of advertising demand based on Peitz and Valletti (2008) (see section 2.2.2). Thus, the second part of the PSB profit function is based on Peitz and Valletti (2008). The way in which the PSB sets the advertising quantity  $a_2$  and the advertising charge  $r_2$  is endogenously determined by the model. The cost of both channels is not part of this analysis and is set to zero for simplicity. The profit function is:

$$\pi_{PSB}(a_2) = a_2 \cdot r_2 + V_{PSB} \cdot f \tag{2.11}$$

The PSB only has the advertising quantity  $a_2$  as strategic variable and is not able to influence the number of viewers  $V_{PSB}$  in this setting which disregards the influence of advertising levels on the viewer utility.

#### Pay-TV

The Pay-TV channel sets an advertising quantity  $a_1$ , and the advertising charge  $r_1$  is endogenously determined by the model. In contrast to the PSB, the Pay-TV channel also sets the subscription fee, s, for its programs. Unlike Peitz and Valletti (2008), where viewers are charged on a per-transaction basis, i.e. pay-per-view payments, the viewers in our model are charged on a lump-sum basis. Similarly to PSB, viewers have to pay a subscription fee, s, once they have decided to subscribe to both, PSB and Pay-TV. As viewers are assumed only to watch Pay-TV once they have subscribed to it in this model, viewers are, here, equal to subscribers.<sup>4</sup>  $\delta$  is defined as the proportion of viewers that subscribe to both, PSB and Pay-TV, and N is defined as the mass of viewers we get for the profit function

$$\pi_{Pay-TV}(a_1, s) = a_1 \cdot r_1 + V_{Pay-TV} \cdot s \tag{2.12}$$

As the marginal subscribers for Pay-TV  $\hat{\mu}_{Pay-TV}$  and for PSB  $\hat{\mu}_{PSB}$  are both determined by the level of s, the Pay-TV channel can influence the number of viewers for the PSB as well as its own channel.

#### 2.2.4 Equilibrium

Equilibrium is the solution of a two-stage game. In stage 1, both channels choose an advertising quantity,  $a_i$ , simultaneously and the Pay-TV channel sets the subscription fee, s. In this simple setting, which excludes the influence of advertising levels on viewer utility, the advertising quantity of one channel does not affect the profit of the other channel. Only the Pay-TV channel's decision regarding s influences the profit of the PSB by affecting the number of viewers. In stage 2, advertisers decide to place adverts on none, one or both channels. Viewers can subscribe to PSB, both PSB and Pay-TV, or to nothing at all. Unlike viewers, however, advertisers are extremely interested in how many viewers are watching their adverts. Hence, markets are related, but not interrelated.

For given advertising levels,  $a_i$ , and given subscription fees, s and f, advertisers and viewers

<sup>&</sup>lt;sup>4</sup>If the number of viewers were not equal to the number of subscribers for Pay-TV, the number of viewers would be relevant for advertising revenues, and the number of subscribers would be relevant for the revenues generated by the subscription fee.

take their decisions in stage 2. Knowing the marginal subscriber  $\hat{\mu}_{Pay-TV} = f + s$  and  $\hat{\mu}_{PSB} = f$  in stage 2, the viewer demand is:

$$V_{Pay-TV} = N(1 - f - s) (2.13)$$

and

$$V_{PSB} = N \cdot s \tag{2.14}$$

Solving (2.9) and (2.10) to the advertising charges,  $r_1$  and  $r_2$ , we get:

$$r_1 = N\delta (1 - a_1) = N(1 - f - s) (1 - a_1)$$
(2.15)

and

$$r_2 = N\phi(1 - a_2) = Ns(1 - a_2) \tag{2.16}$$

for Pay-TV and PSB respectively.

 $\frac{dr_1}{da_1} < 0$  and  $\frac{dr_2}{da_2} < 0$  are true as long as someone subscribes to Pay-TV and PSB respectively. Thus, increasing advertising quantity,  $a_i$ , lowers the advertising charge  $r_i$ .

In stage 1, the PSB only sets an advertising quantity,  $a_2$ . As the PSB cannot influence the volume of viewers in this simple setting, it does not have to take any decisions in stage 2 into account. It simply maximizes its profit function,  $\pi_{PSB}(a_2)$ , with respect to  $a_2$ . The profit maximizing advertising quantity  $a_2$  is:

$$a_2 = \frac{1}{2}. (2.17)$$

In contrast to the PSB, the Pay-TV channel has two strategic variables, the advertising quantity,  $a_1$  and the subscription fee, s. Thus it also takes viewer decisions on stage 2 into account.  $\rho(a_1) \equiv a_1(1-a_1)$  is the advertising revenue per viewer. The profit function (2.12) can be written as:

$$\pi_{Pay-TV}(a_1, s) = V_{Pay-TV}(\rho(a_1) + s)$$
(2.18)

The equilibrium is characterized by the solution of the two partial derivatives

$$\frac{\partial \pi_{Pay-TV}}{\partial a_1} = \frac{\partial \rho(a_1)}{\partial a_1} = 0 \tag{2.19}$$

$$\frac{\partial \pi_{Pay-TV}}{\partial s} = \frac{\partial V_{Pay-TV}}{\partial s} (\rho(a_1) + s) + V_{Pay-TV} = 0$$
 (2.20)

As viewer decisions are independent from advertising quantity, the Pay-TV channel and the PSB set the same advertising quantity,  $a_i$ , in equilibrium.

$$a_1 = a_2 = \frac{1}{2} \tag{2.21}$$

 $\frac{\partial V_{Pay-TV}}{\partial s}$  is negative as a higher subscription fee, s, reduces the number of viewers subscribing Pay-TV. The solution to (2.20) is:

$$s = \frac{1}{2} \left( 1 - f - \rho(a_1) \right) \tag{2.22}$$

Hence, the equilibrium subscription fee for Pay-TV, s, is linear in the advertising revenue per viewer,  $\rho(a_1)$ , and the mandatory broadcasting fee, f. The higher the equilibrium advertising revenue per viewer,  $\rho(a_1)$ , and the higher f, the lower s. If f or  $\rho(a_1)$  were to be increased, the Pay-TV channel would decrease its subscription fee, s, by half of the amount that f or  $\rho(a_1)$  were increased.

Increasing the advertising revenue per viewer has two effects on the Pay-TV channel's profit. Higher advertising revenues per viewer directly increase profits as the Pay-TV channel gets more money from advertisers for every single viewer watching its programs. Hence, it is more important to attract many viewers to watch the programs. Taking this into account, the Pay-TV channel will decrease its subscription fee, s, in order to attract more viewers. The higher number of viewers does not outweigh the lower subscription fee. Thus, the revenues generated by the subscription fee decrease. But lowering the subscription fee, s, is still profitable as long as the positive effect on advertising revenues due to the higher number of viewers remains higher than the negative effect on revenues from lower subscription fees. The total effect on profits is maximized if s is lowered by half of the amount that  $\rho(a_1)$  is increased. Substituting  $a_1 = \frac{1}{2}$ 

for the equilibrium advertising quantity in (2.22) the equilibrium subscription fee s is

$$s = \begin{cases} \frac{3}{8} - \frac{1}{2}f & \text{if } f \le \frac{3}{4} \\ 0 & \text{if } f > \frac{3}{4} \end{cases}$$
 (2.23)

A change in the broadcasting fee influences the profits of the Pay-TV channel as it determines its number of viewers. The proportion of viewers watching Pay-TV is decreased by the increase of f. A lower number of viewers influences both sources of revenues, advertising revenues and revenues generated by subscription fees. This fact can be shown by the first order condition of the profit function with respect to f:

$$\frac{d(\pi_{Pay-TV})}{df} = -N\left(\rho(a_1) + s\right) \tag{2.24}$$

Similar to the change in advertising revenues per viewer, the Pay-TV channel's best response to an increase in f is to decrease its subscription fee, s. This has a positive effect on the number of viewers as long as there is enough room for lowering the subscription fee, s. The profit is maximized if s is decreased by half of the amount that f was increased. For  $f > \frac{3}{4}$ , the Pay-TV channel cannot absorb the negative effect on the number of viewers by decreasing s, as it is already set its subscription fee to zero.

Figure 1 plots the profit of the Pay-TV channel,  $\pi_{Pay-TV}$ , against f in equilibrium.<sup>5</sup>

The profit of the Pay-TV channel rapidly decreases as f is increased affecting its number of viewers. For  $f > \frac{3}{4}$ , the profit function has a constant slope as the Pay-TV channel cannot reduce s if f is increased.

Figure 2 plots the profit of the PSB,  $\pi_{PSB}$ , against f in equilibrium.

Figure 2 shows that the profit of the PSB increases with the subscription fee for PSB, f, for small values of f only. In contrast, for higher values of f profits are decreasing in f. There is no money to make for the PSB for  $f \geq \frac{3}{4}$  as there is no single potential viewer who watches its programs. As f is increased, the PSB faces two effects. Firstly, it receives a higher subscription fee per viewer, that is, an increase in profits. Secondly, its number of viewers decreases determined by the subscription fee, s, for Pay-TV. As previously discussed, the Pay-

 $<sup>^{5}</sup>$ The number of potential viewers N is set to one in this and all following figures.

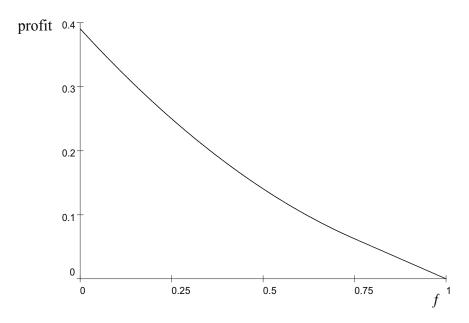


Figure 2-1: Pay-TV channel profit - simple model

TV channel decreases its subscription fee, s, by half the amount that f is increased. The total effect on revenues from subscription fees is positive but the effect on advertising revenues is always negative. For small values of f the positive effect on revenues from subscription fees more than compensates the negative effect on advertising revenues as f is increased. For higher values of f the negative effect more than compensates the positive one. If  $f \geq \frac{3}{4}$ , the subscription fee for Pay-TV g is set to zero (see above), thus no one watches PSB. The Pay-TV switched to a Free-TV channel and viewers are assumed to watch the Pay-TV channel once they have subscribed for both.

#### 2.3 The two-sided market model

In this section an extended model is presented. Based on the simple model, an extended setup is developed in which viewers dislike advertising.<sup>6</sup> A disutility parameter for advertising is introduced in viewing demand. The simple model is, in fact, just a special case of this extended model in which the disutility parameter from advertising is set to zero. The modeling

<sup>&</sup>lt;sup>6</sup>Kohlschein (2005) refers to a survey (see: TV-Spots nerven am meisten. In: Horizont 27, 2001) that asked viewers to assess the level of advertising slots in television. 83.1% of the respondants said that the level of advertising spaces is too high in television.

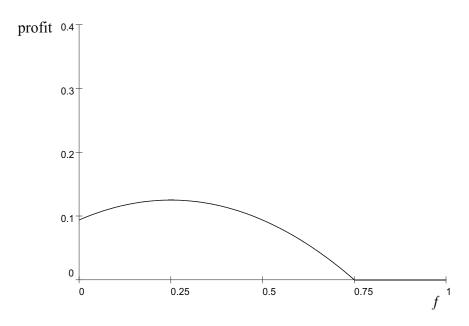


Figure 2-2: Public service broadcaster profit- simple model

of advertisers and TV channels is still valid. Only viewing demand is adjusted. Markets are not only related but interrelated and two-sided. As viewers dislike advertising they are interested in as little advertising as possible when watching TV. Therefore, the amount of advertising is passed on as an indirect charge to consumers. Advertisers exert a negative externality on the programs viewers. Contrastingly, advertisers want to see as many viewers as possible watching their adverts. Thus viewers exert a positive externality on the companies that place adverts within a program. The platforms - i.e. the TV channels - compete for both, viewers and advertisers, and set the subscription fee and advertising quantity in the case of Pay-TV and the advertising quantity in the case of PSB in order to maximize their profits.

#### 2.3.1 Viewers

As presented in the simple model, a viewer has an individual budget,  $\mu \in [0, 1]$ , for watching TV, the broadcasting fee and the subscription fee for Pay-TV are f and  $s \in [0, 1]$  respectively. Viewers decide between subscribing to nothing, subscribing to PSB and subscribing to both, PSB and Pay-TV. Again, they watch only Pay-TV if they have subscribed to both. Now, however, viewers dislike advertising.  $\gamma \in [0, 1]$  denotes the disutility parameter from advertising.  $\gamma$  is assumed to be identical across all viewers. From a viewer's point of view,  $\gamma$ , as with f and

s, is an indirect charge that burdens their individual budget,  $\mu$ . Advertising on Pay-TV and PSB burdens a viewer's budget by  $\gamma a_1$  and  $\gamma a_2$  respectively. Hence, an individual consumer makes his viewing decision according to the following allocation rule:

$$d_{PSB}(\mu) = \begin{cases} 0 & \text{if } \mu \ge f + s + \gamma a_1 \lor \mu < f + \gamma a_2 \\ 1 & \text{if } \mu \le f + s + \gamma a_1 \end{cases}$$
 (2.25)

$$d_{Pay-TV}(\mu) = \begin{cases} 0 & \text{if } \mu \le f + s + \gamma a_1 \\ 1 & \text{if } \mu \ge f + s + \gamma a_1 \end{cases}$$
 (2.26)

Therefore, the marginal subscriber to Pay-TV is  $\hat{\mu}_{Pay-TV} = f + s + \gamma a_1$  and the marginal subscriber for PSB is  $\hat{\mu}_{PSB} = f + \gamma a_2$  where  $\hat{\mu}_{Pay-TV}$  and  $\hat{\mu}_{PSB} \in [0,1]$ .  $s + \gamma a_1 \ge \gamma a_2$  is assumed to be true.

For the proportion of potential viewers that subscribe to both  $\delta$  we get

$$\delta = 1 - \hat{\mu}_{Pay-TV} = 1 - f - s - \gamma a_1 \tag{2.27}$$

and for the proportion of viewers for PSB  $\phi$  we get

$$\phi = \hat{\mu}_{Pay-TV} - \hat{\mu}_{PSB} = s + \gamma (a_1 - a_2) \tag{2.28}$$

Condition  $s + \gamma a_1 \ge \gamma a_2$  is the same as  $\phi \ge 0$ . Hence, the condition is always fulfilled as long as we do not allow for a negative market share for PSB.

The proportion of potential viewers who subscribe to nothing is  $\hat{\mu}_{PSB} = f + \gamma a_2$ . Hence, the advertising quantity set by the PSB determines the proportion of viewers for PSB  $\phi$ , and the proportion of viewers who exit the market.  $\phi$  positively depends on the advertising quantity on Pay-TV  $a_1$ . In contrast, the proportion of viewers for Pay-TV  $\delta$  is independent from the advertising quantity of the other channel  $a_2$ .

Accordingly the viewing demand functions for the channels are equal to that of the simple model:

$$V_{Pay-TV} = N\delta \tag{2.29}$$

and

$$V_{PSB} = N\phi \tag{2.30}$$

where  $V_{Pay-TV}$  and  $V_{PSB}$  denote the number of Pay-TV and PSB viewers respectively.

#### 2.3.2 Equilibrium

Advertisers and viewers make their decisions in stage 2 for given subscription fees s and f. Knowing the marginal subscribers,  $\hat{\mu}_{Pay-TV} = f + s + \gamma a_1$  and  $\hat{\mu}_{PSB} = f + \gamma a_2$  in stage 2, viewer demand is adjusted:

$$V_{Pay-TV} = N(1 - f - s - \gamma a_1)$$
(2.31)

and

$$V_{PSB} = N(s + \gamma(a_1 - a_2)) \tag{2.32}$$

Solving (2.9) and (2.10) to the advertising charges  $r_1$  and  $r_2$  we get

$$r_1 = N\delta(1 - a_1) = N(1 - f - s - \gamma a_1)(1 - a_1)$$
(2.33)

and

$$r_2 = N\phi(1 - a_2) = N(s + \gamma(a_1 - a_2))(1 - a_2)$$
(2.34)

for Pay-TV and PSB respectively.

 $\frac{dr_1}{da_1} < 0$  and  $\frac{dr_2}{da_2} < 0$  and  $\frac{dr_2}{da_1} > 0$  are true as long as someone subscribes to Pay-TV and PSB respectively. Thus, increasing advertising quantity,  $a_i$ , lowers the advertising charge,  $r_i$ , and increasing advertising quantity on Pay-TV,  $a_1$ , lowers the advertising charge on PSB.

In stage 1, the Pay-TV channel sets advertising quantity,  $a_1$ , and subscription fee, s. Now, viewer decisions are taken into account in stage 2 for setting s as well as  $a_1$ . The advertising revenue per viewer is defined as  $\rho(a_1) \equiv a_1(1-a_1)$ . The profit function introduced in the simple model is still valid. Only the number of viewers  $V_{Pay-TV}$  is adjusted.

$$\pi_{Pay-TV}(a_1, s) = V_{Pay-TV}(\rho(a_1) + s)$$
(2.35)

As in the simple model, equilibrium is characterized by the solution of two partial derivatives.

$$\frac{\partial \pi_{Pay-TV}}{\partial a_1} = \frac{\partial V_{Pay-TV}}{\partial a_1} (\rho(a_1) + s) + \frac{\partial \rho(a_1)}{\partial a_1} V_{Pay-TV} = 0$$
 (2.36)

$$\frac{\partial \pi_{Pay-TV}}{\partial s} = \frac{\partial V_{Pay-TV}}{\partial s} (\rho(a_1) + s) + V_{Pay-TV} = 0$$
 (2.37)

The Pay-TV channel faces two direct effects when setting  $a_1$ . Firstly, the number of viewers decreases as  $a_1$  is increased. The effect is stronger the more potential viewers are bothered by adverts:  $\frac{\partial V_{Pay-TV}}{\partial a_1} = -\gamma < 0$ . In the simple model this effect is not relevant as  $\gamma$  is set to zero. Secondly, the advertising revenues per viewer,  $\rho(a_1)$ , changes as  $a_1$  is increased. For  $a_1 < \frac{1}{2}$ , there is an increase in  $\rho(a_1)$  if  $a_1$  is increased, and for  $a_1 > \frac{1}{2}$  the advertising revenues per viewer,  $\rho(a_1)$ , decreases as  $a_1$  is increased. Therefore,  $a_1 = \frac{1}{2}$  in equilibrium in the simple model.

We get only one solution for  $f \in [0,1]$  and  $\gamma \in [0,1]$  in equilibrium.

$$a_1 = \frac{1}{2}(1 - \gamma) \tag{2.38}$$

 $a_1$  is linear in the nuisance parameter,  $\gamma$ , and depends on  $\gamma$  negatively as it is expected intuitively. For the subscription fee, s, we get in equilibrium:

$$s = \begin{cases} \frac{1}{8} \left( 3\gamma^2 - 2\gamma - 4f + 3 \right) & \text{if} \quad f \le \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma \\ 0 & \text{if} \quad f > \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma \end{cases}$$
 (2.39)

**Proposition 1** The equilibrium values derived for  $a_1$  and s constitute a maximum.

In stage 1, the PSB sets the advertising quantity,  $a_2$ . In contrast to the simple model, the PSB can now influence its volume of viewers as the nuisance factor caused by advertising is introduced. The higher its advertising quantity,  $a_2$ , the less viewers will be attracted to its

programming schedule. Hence, viewers' decisions in stage 2 will be taken into account. As with Pay-TV, the profit function introduced in the simple model is still valid. Only the number of viewers,  $V_{PSB}$ , is adjusted.

$$\pi_{PSB}(a_2) = V_{PSB}(a_2(1 - a_2) + f) \tag{2.40}$$

It maximizes its profit function,  $\pi_{PSB}(a_2)$ , with respect to  $a_2$ . The profit maximizing advertising quantity depends on s and  $a_1$ , which are set by the Pay-TV channel. In equilibrium we get after substituting for s and  $a_1$  for the maximum:

$$a_{2} = \begin{cases} \frac{1}{2} & \text{if} \quad \gamma = 0 \\ \frac{1}{\gamma} \left( \frac{5}{12} \gamma - \frac{1}{6} f - \frac{1}{24} \gamma^{2} + \frac{1}{8} \right) & \text{if} \quad \gamma > 0 \\ -\frac{1}{3} \sqrt{\frac{1}{4} f \gamma - \frac{3}{16} \gamma - \frac{3}{8} f + \frac{1}{4} f^{2} + \frac{23}{32} \gamma^{2}} \\ + \frac{1}{16} \gamma^{3} + \frac{1}{64} \gamma^{4} + \frac{25}{8} f \gamma^{2} + \frac{9}{64} \end{cases} ) & \text{if} \quad \gamma > 0 \\ 0 & \text{if} \quad \wedge f \leq \frac{1}{4(2\gamma+1)} \left( 2\gamma - \gamma^{2} + 3 \right) \\ \wedge f > \frac{1}{4(2\gamma+1)} \left( 2\gamma - \gamma^{2} + 3 \right) \end{cases}$$

**Proposition 2** There is only one solution for  $a_2$  that constitutes a maximum.

The interpretations presented in the simple model are still valid in general. However, introducing the nuisance parameter,  $\gamma$ , changes critical values as TV-channels have to take viewers' decisions into account when setting  $a_i$ . In the general model, the PSB also takes the decision of the Pay-TV-channel on  $a_1$  into account when setting its own advertising quantity,  $a_2$ . The effect of an increase in  $a_1$  on the profit of the PSB can be illustrated by analyzing its profit function.

$$\pi_{PSB} = V_{PSB}(a_2(1 - a_2) + f) = N(s + \gamma(a_1 - a_2))(a_2(1 - a_2) + f)$$
(2.42)

There are two effects on  $\pi_{PSB}$  as  $a_1$  is increased. The spread between the advertising quantities,  $a_1$  and  $a_2$ , directly influences the number of viewers,  $V_{PSB}$ . The higher the spread between  $a_1$  and  $a_2$ , the more people are likely to watch PSB as it is expected intuitively. The

effect is stronger if viewers have a strong aversion to advertising. There is also an indirect effect caused by a change in the subscription fee for Pay-TV, s. Depending on the level of advertising quantity,  $a_1$ , and the nuisance parameter,  $\gamma$ , s is either increased or decreased as  $a_1$  is increased.

To analyze the effect of the nuisance level caused by advertising on the decision making of the TV-channels, market outcomes are compared for different values of  $\gamma$ . In addition to  $\gamma=0$ , i.e. no irritation caused by advertising as presented in the simple model, we consider the case of  $\gamma=1$ , i.e. maximum irritation caused by advertising and the case of  $\gamma=\frac{1}{2}$  in the following. Figure 3 plots the profit of the Pay-TV channel  $\pi_{Pay-TV}$  against f in equilibrium for  $\gamma=0$ ,  $\gamma=\frac{1}{2}$  and  $\gamma=1$ .

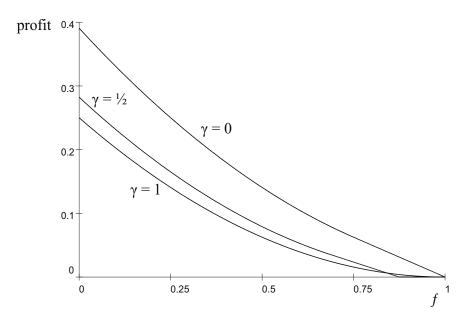


Figure 2-3: Pay-TV channel profit - different values for  $\gamma$ 

Profits decrease in the nuisance parameter  $\gamma$  as it is expected intuitively. The higher  $\gamma$ , the lower the equilibrium advertising quantity of the Pay-TV channel,  $a_1$ . If  $\gamma$  is relatively high the Pay-TV channel reduces its advertising quantity when compared to a setting with viewers that are not bothered by advertising. If not it would loose viewers, thus decreasing both sources of revenues, advertising revenue and revenues generated by subscription fees. Advertisers would pay less as less viewers watched their adverts. For maximum amount of nuisance caused by advertising, i.e.  $\gamma = 1$ , the Pay-TV channel does not show any adverts,  $(a_1 = 0)$ , and is financed

by the subscription fee, s, only. For a given nuisance parameter,  $\gamma$ , the advertising quantity is a constant as it is described by (2.38). As f is increased, the Pay-TV channel reduces its subscription fee, s, to compensate the increased charge on viewers' budget, thus decreasing the number of viewers to the Pay-TV channel. For  $\gamma = \frac{1}{2}$ , the Pay-TV channel does not make any profits for  $f > \frac{7}{8}$ .

Figure 4 plots the profit of the PSB,  $\pi_{PSB}$ , against f in equilibrium for  $\gamma = 0$ ,  $\gamma = \frac{1}{2}$  and  $\gamma = 1$ .

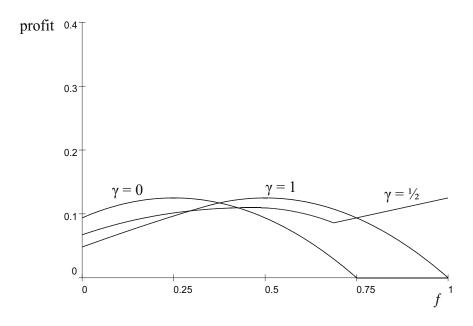


Figure 2-4: Public service broadcaster profit - different values for  $\gamma$ 

Depending on the nuisance parameter,  $\gamma$ , the PSB prefers a high or low subscription fee, f, given exogenously. If viewers are not bothered by adverts, the PSB prefers a relatively low subscription fee, f. It maximizes profits with  $f = \frac{1}{4}$ . For  $\gamma = 1$  it maximizes profits with  $f = \frac{1}{2}$  and for  $\gamma = \frac{1}{2}$  it maximizes profits with f = 1.

If viewers have a strong aversion to adverts, i.e.  $\gamma = 1$ , the PSB does not show any adverts for  $f > \frac{1}{3}$ . Hence, it cannot decrease advertising quantity,  $a_2$ , if f is increased to compensate viewers. Interestingly, profits increase for  $\triangle f > 0$  and  $\frac{1}{3} \le f \le \frac{1}{2}$  even though advertising quantity,  $a_2$ , is already set to zero. For  $f > \frac{1}{3}$  and  $\gamma = 1$  there are no adverts on either channel,

i.e.  $a_1 = a_2 = 0$ . The profit of the PSB can be described as

$$\pi_{PSB} = Nsf = N\frac{1}{2}(1-f)f \tag{2.43}$$

Thus, profits only depend on fees, s and f. The PSB faces two effects as f is increased. A negative effect in which fewer viewers watch its program as the subscription fee for Pay-TV, s, is decreased, which determines the number of viewers watching PSB and a positive effect in which the PSB gets more money per viewer from subscription fees as f is increased. For  $\frac{1}{3} \leq f \leq \frac{1}{2}$ , the net of both effects is positive. For  $f > \frac{1}{2}$ , the net of both effects is negative and profits decrease.

If the nuisance parameter,  $\gamma$ , is  $\frac{1}{2}$ , the PSB does not show any adverts for  $f \geq \frac{15}{32}$ , i.e.  $a_2 = 0$ . For  $f \geq \frac{11}{16}$  the Pay-TV channel sets its subscription fee, s = 0, and switches to a Free-TV channel. As the advertising quantity of the Pay-TV channel,  $a_1$ , is a constant,  $(a_1 = \frac{1}{4})$ , the profit function of the PSB is reduced to  $\pi_{PSB} = N\frac{1}{8}f$ . Thus it is linear in f.

#### 2.4 The Game with Possible Market Exit

In the preceding analysis fixed cost are not taken into account. In this chapter we want to introduce fixed cost to the profit function of the Pay-TV channel and find critical values for f where it is not profitable for the Pay-TV channel to remain in the market or enter the market. We then develop equilibrium values for a setting with no competitor for the public broadcaster, i.e. no Pay-TV channel in the market. Next we analyze which setting a PSB would prefer, a scenario with a Pay-TV channel as competitor and a relatively low broadcasting fee, f, or a scenario in which it exists as a monopolist with a relatively high broadcasting fee, f. After that a third stage is introduced in the model where the government maximizing the number of viewers for PSB sets the broadcasting fee, f.

#### 2.4.1 The Adjusted Model

In the profit function of the Pay-TV channel, fixed cost, I, are introduced.

$$\pi_{Pay-TV} = V_{Pay-TV}(\rho(a_1) + s) - I \tag{2.44}$$

To make interpretations possible, an example with I = 0.05N is considered. As defined in

the simple model, N is the number of potential viewers, thus constituting the viewer market. Revenues have to exceed fixed cost in order to make it profitable to either remain in or enter the market:

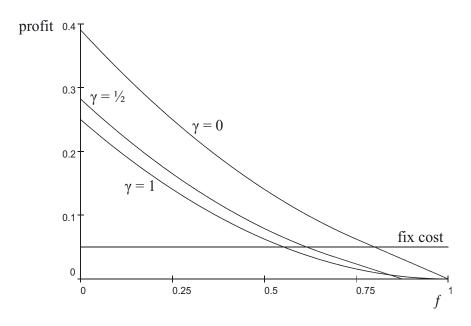


Figure 2-5: Pay-TV channel profit - fix cost introduced

Figure 5 shows that the Pay-TV channel runs a profitable business as long as f < 0.553 for  $\gamma = 1$ , f < 0.615 for  $\gamma = \frac{1}{2}$  and f < 0.8 for  $\gamma = 0$ .

The PSB is already in the market. It maximizes profits to generate political goodwill. However, fixed cost are assumed not to be relevant as political authorities are supportive of a PSB and therefore unlikely to shut it down. The PSB faces different viewing demands if it does not compete with a Pay-TV channel for viewers. If there is no Pay-TV channel, an individual consumer makes his viewing decision according to the following allocation rule:

$$d_{PSB}(\mu) = \begin{cases} 0 & \text{if } \mu < f + \gamma a_2 \\ 1 & \text{if } \mu \ge f + \gamma a_2 \end{cases}$$
 (2.45)

The proportion of potential viewers who subscribe to PSB  $\phi$  is:

$$\phi = 1 - f - \gamma a_2 \tag{2.46}$$

The other proportion of potential viewers do not subscribe to PSB. The profit function of the PSB is

$$\pi_{PSB}(a_2) = V_{PSB}(a_2(1 - a_2) + f) = N(1 - f - \gamma a_2)(a_2(1 - a_2) + f)$$
(2.47)

The profit maximizing advertising quantity is

$$a_{2} = \begin{cases} \frac{1}{3\gamma} \left( \gamma - f - \sqrt{f\gamma - \gamma - 2f + f^{2} + \gamma^{2} + 3f\gamma^{2} + 1} + 1 \right) & \text{if } \gamma > 0 \\ \frac{1}{2} & \text{if } \gamma = 0 \end{cases}$$
 (2.48)

For  $\gamma = 0$ , viewing demand is independent from advertising quantity. Hence, the advertising quantity,  $a_2 = \frac{1}{2}$ , is equal to the simple model. However, profits change as viewer demand changes.

Figure 6 shows the adjusted profit of the PSB for  $\gamma=0,\,\gamma=\frac{1}{2}$  and  $\gamma=1$  taking critical values for the market exit of Pay-TV into account.

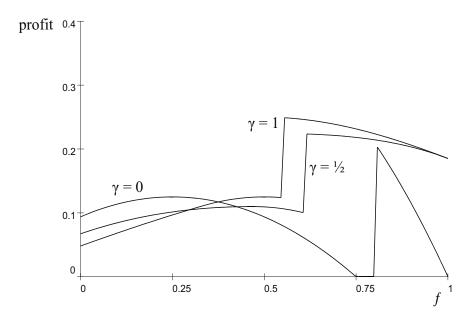


Figure 2-6: Public service broadcaster profit - possible market exit

The PSB is always better off if it can act as a monopolist but faces a higher broadcasting fee, f, for  $\gamma = \frac{1}{2}$  and  $\gamma = 1$ . In the case in which no nuisance is caused by advertising, it only makes less profits for very high values of f compared to the profit maximizing value for f

 $(f = \frac{1}{4})$  in the case of competition against Pay-TV.

#### 2.4.2 Maximizing Viewing Demand for PSB

To answer the question of which setting is more likely in this model, a PSB as a monopolist or competition between Pay-TV and PSB, a third stage is introduced to the adjusted model. Prior to the game analyzed so far, the government sets broadcasting fee, f. It is assumed that the government maximizes the number of viewers for PSB. The government has all available information, i.e. it takes decisions in stage 2 and 3 into account. For  $\gamma = \frac{1}{2}$  and  $\gamma = 1$ , the number of viewers watching PSB  $V_{PSB}$  is maximized if the broadcasting fee, f, has the value where the Pay-TV channel exits the market. A further increase in f reduces  $V_{PSB}$ . This result can be expected intuitively as a high number of viewers change from Pay-TV to PSB if the Pay-TV channel exits the market. Hence, the government maximizing the number of viewers watching PSB selects the value for f where the profit of the PSB is maximized, too. For  $\gamma = 0$ , in which no nuisance is caused by advertising, the government sets f = 0. As the number of viewers for PSB  $V_{PSB}$  only depends on the subscription fee, s, which decreases in f, this result could be expected, too. Thus, there are more viewers watching PSB with f = 0 and a Pay-TV channel in the market than with a high broadcasting fee, f, and no competing Pay-TV channel.

#### 2.5 Discussion and Extensions

In this section we discuss some of our modeling assumptions and possible extensions and modifications.

Viewer behavior. We assumed that viewers watch only Pay-TV once they have decided to subscribe to it. A different allocation of viewing time would be more realistic. As the subscription fee for Pay-TV is charged on a lump-sum basis, viewer decision making could be modeled in two stages. In stage 1, viewers decide on the subscription to Pay-TV and in stage 2 viewers allocate their viewing time.

The portfolio approach incorporates crucial assumptions about the behavior of viewers. An empirical analysis asking viewers what determines their decision making would be interesting. Maybe, the portfolio approach and the standard approach of maximizing utility could be synthesized in an integrated approach.

Although the broadcasting fee is mandatory for every TV owner, there are viewers who

watch TV and illegally do not pay the broadcasting fee. It is interesting how the size of this group depends on the level of the broadcasting fee.

**Program content.** In contrast to most papers analyzing competition in television markets, we do not model different program contents explicitly. Different contents could be taken into account when modeling viewer demand. Additionally, broadcasters could invest in a higher quality of programming. Incentives are probably higher for Pay-TV as the cost can be shifted on to viewers.

Advertisers. Following a large part of the literature, advertisers can only reach a particular viewer via a particular TV channel. In practice, advertisers face different advertising channels to reach viewers. Thus, TV channels compete with other advertising channels such as newspapers for advertisers. Taking this into account there is competition for advertisers and therefore advertising prices decrease.

**Dynamic approach.** The comparative-static analysis on market exit of the Pay-TV channel could be extended to a dynamic setting. A PSB would possibly change its behavior in period one to squeeze the Pay-TV channel out of the market. Accordingly, it would accept lower profits in period one in order to hold a monopoly in the following periods.

**Regulator.** The objectives of the government setting the broadcasting fee are ambiguous. In our model it is assumed that it maximizes the number of viewers watching PSB. It is interesting how this setting compares to a setting with a regulator maximizing welfare.

**Input markets.** The analysis of effects related to market exit of the Pay-TV channel could be extended to input markets. If there is only one TV channel it could exploit its market power and enforce lower prices for input factors.

Oligopoly setting. A model with one Pay-TV channel and one PSB does not reflect the current European TV broadcasting market. Introducing a free-to-air channel as a third player would be very interesting as there would be three players with different strategic variables. Commercial broadcasters could also decide in the first stage between running a Pay-TV channel or a free-to-air channel and could then compete with one PSB and a second channel, either a Pay-TV channel or a free-to-air channel.

Welfare. A welfare analysis is missing in this work. It would be especially interesting to see if the market exit of a Pay-TV channel would really reduce welfare as competition was decreased. In terms of viewers it seems ambiguous as to whether the standard calculation of consumer surplus is in line with a portfolio approach not based on the maximization of utility.

As a two-sided market approach is, in general, associated with strong computational problems due to inter-market network effects a model could only be extended with caution. Additionally, it becomes difficult to identify effects when taking many different aspects into account.

#### 2.6 Summary and Conclusion

This work has analyzed the behavior of two competing TV channels - Pay-TV and PSB assuming that consumers engage in mental accounting and develop a portfolio demand. A crucial role in our analysis has been played by the 'nuisance factor' associated with advertising and the broadcasting fee that has to be paid by both viewers of Pay-TV and PSB.

The greater the nuisance associated with advertising, the less adverts are shown on Pay-TV, which results in decreasing profits. If viewers display a strong aversion to advertising, the Pay-TV channel decides not to show any adverts at all. A high broadcasting fee considerably decreases the Pay-TV channel's profit. The Pay-TV channel tries to compensate viewers facing a high broadcasting fee by reducing its own subscription fee. If the broadcasting fee is sufficiently high the Pay-TV channel does not charge viewers and therefore switches to a free-to-air channel. However, the broadcasting fee does not affect the level of advertising on Pay-TV.

The PSB can only decide on its level of advertising. If viewers have to pay an increased charge for PSB they are compensated with fewer commercials on this channel. If that charge is sufficiently enough the PSB does not show any adverts at all. The greater the nuisance associated with adverts, the higher the broadcasting fee the PSB prefers for its own program.

Taking fixed cost for Pay-TV into account, the Pay-TV channel exits the market if the broadcasting fee is too high. As long as viewers object to advertising the PSB prefers a monopoly setting. If the government maximizes the number of PSB viewers the broadcasting fee is set so that the Pay-TV channel leaves the market and profits of the PSB are maximized.

From the point of view of competition policy the last result is concerning. The government sets the broadcasting fee that the PSB would set for itself. The PSB would set this broadcasting

fee to squeeze the Pay-TV channel out of the market, thus facing no competition. Negative effects on viewers are likely in this scenario. Additionally, there could be negative effects on input markets due to the high market power of the PSB.

#### 2.A Appendix

**Proof of Proposition 1.** The critical values of the Pay-TV channel profit function  $\pi_{Pay-TV}$  are

$$a_1 = \frac{1}{2}(1 - \gamma) \tag{2.49}$$

and

$$s = \begin{cases} \frac{1}{8} \left( 3\gamma^2 - 2\gamma - 4f + 3 \right) & \text{if} \quad f \le \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma \\ 0 & \text{if} \quad f > \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma \end{cases}$$
 (2.50)

The sufficient condition for a maximum is fulfilled if the eigenvalues of the Hessian matrix are all positive at the stationary point, i.e. the Hessian is negative definite. For the Hessian matrix of  $\pi_{Pay-TV}$  we get

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 \pi_{Pay-TV}}{\partial s^2} & \frac{\partial^2 \pi_{Pay-TV}}{\partial a_1 \partial s} \\ \frac{\partial^2 \pi_{Pay-TV}}{\partial s \partial a_1} & \frac{\partial^2 \pi_{Pay-TV}}{\partial a_1^2} \end{pmatrix} = \begin{pmatrix} -2 & 2a_1 - \gamma - 1 \\ 2a_1 - \gamma - 1 & 2\left(f + s - \gamma + 3\gamma a_1 - 1\right) \end{pmatrix}$$
(2.51)

Substituting for  $a_1$  and s the Hessian is for  $f \leq \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma$ 

$$\mathbf{H} = \begin{pmatrix} -2 & -2\gamma \\ -2\gamma & -\frac{1}{4} \left( 9\gamma^2 - 2\gamma - 4f + 5 \right) \end{pmatrix}$$
 (2.52)

The eigenvalues are

$$\lambda_{1,2} = \frac{1}{2}f + \frac{1}{4}\gamma - \frac{9}{8}\gamma^2 - \frac{13}{8}$$

$$\pm \frac{1}{8}\sqrt{24f + 12\gamma + 16f\gamma + 16f^2 + 206\gamma^2 - 36\gamma^3 + 81\gamma^4 - 72f\gamma^2 + 9}$$
(2.53)

 $\lambda_1$  and  $\lambda_2$  are always negative for  $\gamma \in [0,1]$ ,  $f \in [0,1]$  and  $f \leq \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma$ , i.e. the Hessian is negative definite.

Substituting for  $a_1$  and s the Hessian is for  $f > \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma$ 

$$\mathbf{H} = \begin{pmatrix} -2 & -2\gamma \\ -2\gamma & -\left(3\gamma^2 - \gamma - 2f + 2\right) \end{pmatrix} \tag{2.54}$$

The eigenvalues are

$$\lambda_{1,2} = f + \frac{1}{2}\gamma - \frac{3}{2}\gamma^2 - 2$$

$$\pm \frac{1}{2}\sqrt{4f\gamma + 4f^2 + 17\gamma^2 - 6\gamma^3 + 9\gamma^4 - 12f\gamma^2}$$
(2.55)

 $\lambda_1$  and  $\lambda_2$  are always negative for  $\gamma \in [0,1]$ ,  $f \in [0,1[$  and  $f > \frac{3}{4} + \frac{3}{4}\gamma^2 - \frac{1}{2}\gamma$ , i.e. the Hessian is negative definite. For  $\gamma = 1$  and f = 1,  $\lambda_1 = 0$ , i.e. the Hessian is negative semidefinite. Thus, the sufficient condition for a maximum is fulfilled for  $\gamma \in [0,1]$ ,  $f \in [0,1[$ .

**Proof of Proposition 2.** The critical values of the PSB profit function  $\pi_{PSB}$  are

$$a_{2} = \begin{cases} \frac{1}{2} & \text{if } \gamma = 0 \\ \frac{1}{\gamma} \left( \frac{5}{12} \gamma - \frac{1}{6} f - \frac{1}{24} \gamma^{2} + \frac{1}{8} \right) & \text{if } \gamma > 0 \\ \pm \frac{1}{3} \sqrt{\frac{\frac{1}{4} f \gamma - \frac{3}{16} \gamma - \frac{3}{8} f + \frac{1}{4} f^{2} + \frac{23}{32} \gamma^{2}}{+ \frac{1}{16} \gamma^{3} + \frac{1}{64} \gamma^{4} + \frac{25}{8} f \gamma^{2} + \frac{9}{64}}} \right) & \text{if } \gamma > 0 \\ 0 & \text{if } \gamma > 0 \\ 0 & \text{if } \gamma > 0 \\ \wedge f > \frac{1}{4(2\gamma + 1)} \left( 2\gamma - \gamma^{2} + 3 \right) \end{cases}$$
(2.56)

The sufficient condition for a maximum is fulfilled if the second derivate of the profit function  $\pi_{PSB}$  is negative at the stationary point . For the second derivate we get after substituting for equilibrium values for  $a_1$  and s

$$\frac{d^2 \pi_{PSB}}{da_2^2} = \begin{cases}
\frac{1}{4} N \left( 4f - 10\gamma + 24\gamma a_2 + \gamma^2 - 3 \right) & \text{if } f < -\frac{1}{2}\gamma + \frac{3}{4}\gamma^2 + \frac{3}{4} \\
\gamma N \left( \gamma + 6a_2 - 3 \right) & \text{if } -\frac{1}{2}\gamma + \frac{3}{4}\gamma^2 + \frac{3}{4} < f
\end{cases}$$
(2.57)

For  $\gamma = 0$  and  $a_2 = \frac{1}{2}$  we get

$$\frac{d^2 \pi_{PSB}}{da_2^2} = \begin{cases}
-\frac{1}{4}N(3 - 4f) & \text{if } f \leq \frac{3}{4} \\
0 & \text{if } \frac{3}{4} < f
\end{cases}$$
(2.58)

Thus,  $\frac{d^2\pi_{PSB}}{da_2^2} < 0$  for  $f < \frac{3}{4}$  and the sufficient condition for a maximum is fulfilled. For  $f \geq \frac{3}{4}$  the second derivative test says nothing about the critical point.

For 
$$\gamma > 0$$
,  $f \le \frac{1}{4(2\gamma+1)} \left(2\gamma - \gamma^2 + 3\right)$  and  $a_2 = \frac{1}{\gamma} \left(\frac{5}{12}\gamma - \frac{1}{6}f - \frac{1}{24}\gamma^2 + \frac{1}{8}\gamma^3 + \frac{1}{4}\gamma^4 - \frac{3}{16}\gamma - \frac{3}{8}f + \frac{1}{4}f^2 + \frac{23}{32}\gamma^2 + \frac{1}{16}\gamma^3 + \frac{1}{64}\gamma^4 + \frac{25}{8}f\gamma^2 + \frac{9}{64}\right)$  we get

$$\frac{d^2\pi_{PSB}}{da_2^2} = 2N\sqrt{-\frac{3}{8}f - \frac{3}{16}\gamma + \frac{1}{4}f\gamma + \frac{1}{4}f^2 + \frac{23}{32}\gamma^2 + \frac{1}{16}\gamma^3 + \frac{1}{64}\gamma^4 + \frac{25}{8}f\gamma^2 + \frac{9}{64}}$$
 (2.59)

Hence,  $\frac{d^2\pi_{PSB}}{da_2^2} > 0$  is always true and the critical point substituted for  $a_2$  is a minimum.

For 
$$\gamma > 0$$
,  $f \le \frac{1}{4(2\gamma+1)} \left(2\gamma - \gamma^2 + 3\right)$  and  $a_2 = \frac{1}{\gamma} \left(\frac{5}{12}\gamma - \frac{1}{6}f - \frac{1}{24}\gamma^2 + \frac{1}{8}\gamma^3 + \frac{1}{4}\gamma^4 - \frac{3}{16}\gamma - \frac{3}{8}f + \frac{1}{4}f^2 + \frac{23}{32}\gamma^2 + \frac{1}{16}\gamma^3 + \frac{1}{64}\gamma^4 + \frac{25}{8}f\gamma^2 + \frac{9}{64}\right)$  we get

$$\frac{d^2\pi_{PSB}}{da_2^2} = -2N\sqrt{-\frac{3}{8}f - \frac{3}{16}\gamma + \frac{1}{4}f\gamma + \frac{1}{4}f^2 + \frac{23}{32}\gamma^2 + \frac{1}{16}\gamma^3 + \frac{1}{64}\gamma^4 + \frac{25}{8}f\gamma^2 + \frac{9}{64}}$$
 (2.60)

Thus,  $\frac{d^2\pi_{PSB}}{da_2^2} < 0$  is always true and the critical point substituted for  $a_2$  is a maximum.

For  $\gamma > 0$ ,  $f > \frac{1}{4(2\gamma+1)} (2\gamma - \gamma^2 + 3)$  and  $a_2 = 0$  the second derivative test says nothing about the critical point.

## Chapter 3

# Endogenous Merger Formation and Incentives to Invest in Cost Reducing Innovations

#### 3.1 Introduction

In principle, firms can grow in two different ways. They can either grow internally (i.e., through successful innovations and consequent product market success) or externally (i.e., through mergers and acquisitions). While almost always since Schumpeter (1934) there have been policy debates and academic research on the relationship between market structure and innovation, the relationship between innovation and merger activity has only really grabbed the attention of policy makers, antitrust agencies and academic researchers over the last two decades. While over the last twenty years a rather broad empirical literature has grown on the relationship between mergers and innovation the theoretical literature is still relatively thin. As Schulz (2008) notes in a review of the literature on the impact of mergers on innovations there is still a lack of theoretical papers on this issue: "At the beginning of 2006 an extensive search of the existing literature has not revealed a single theoretical contribution that deals directly with the subject..." (Schulz, 2008, p. 2). This may be somewhat surprising since external and internal

<sup>&</sup>lt;sup>1</sup>For a review of the literature on mergers and innovations and its implications for antitrust policy see Katz and Shelanski (2007).

firm growth strategies are not independent from another, but clearly interrelated, as mergers certainly affect the innovation incentives and activities of both the merged entity and its rivals, while innovation activities can also affect merger formation. In fact, almost all of the papers that analyze the relationship between mergers and innovation focus almost exclusively on the impact that mergers have on innovations, taking mergers as exogenous (see Kleer, 2006; Jost and van der Velden, 2006). And while some of the empirical literature notes that an endogeneity problem may exist, the theoretical literature is largely silent on the question how innovations affect merger *incentives*. This paper jumps into this gap and aims at shedding some light on the question how mergers are endogenously affected by innovations.

For this purpose, this paper analyzes an endogenous merger model that incorporates innovation incentives. We assume that firms are heterogenous in their innovation capability, as
differences in innovation capability have been named an important reason for mergers in the
literature. The idea is that the transfer of knowledge within the merged firm can improve the
innovation capability of the entire entity. Hence, our paper differs from existing theoretical
papers in two key aspects: Firstly, we do not only analyze how mergers affect innovation, but
also how innovation capabilities affect mergers, using an endogenous merger model. Secondly,
we assume that firms are not homogenous, but that they differ in their innovation capability.
For this paper, we confine the analysis to process innovation, i.e., we focus on investments in
marginal cost reductions.

Our paper is related to two other papers that focus on the relationship between mergers and innovations: Kleer (2006) analyzes the impact that horizontal mergers have on process innovation incentives in a Cournot market with three firms. In contrast to our paper Kleer (2006) does not endogenize merger decisions, but rather compares exogenously given market structures. Furthermore, firms are symmetric in their ability to conduct process innovations in his paper. The main result is that - in contrast to the well-known merger paradox of Salant et al. (1983)<sup>2</sup> - most mergers are profitable, as mergers reduce the R&D intensity in the market.

A similar result is obtained by Jost and van der Velden (2006) who model mergers in the

<sup>&</sup>lt;sup>2</sup>Salant et al. (1983) show that in a homogenous product Cournot oligopoly with linear demand and cost mergers between an exogenously given number of firms are only profitable if at least 80% of the firms are involved. The robustness of this result was challenged in the supervening literature (e.g., Deneckere and Davidson, 1985; Perry and Porter, 1985; Farrel and Shapiro, 1990).

context of a patent contest model focusing on synergies. Their exogenous merger model focuses on the innovation market, as the winner of the patent race obtains a monopoly position, i.e., the focus is on drastic innovations. In contrast, we focus on gradual productivity improvements in an endogenous merger setting.

The literature on endogenous mergers offers at least three different approaches to analyze merger patterns (see Horn and Persson, 2001a). In Kamien and Zang (1990, 1991) and Gaudet and Salant (1992) firms simultaneously bid for other firms and ask prices for the own firm. As moves are simultaneous, the approach does not allow for any negotiations between firms or counter-offers which may be more appropriate to assume for merger situations with a limited number of firms involved.

Consequently, the other two approaches have been developed to reflect some kind of bargaining process to form mergers. The second approach also treats merger formation as a non-cooperative game, but with sequential offers. For examples Chatterjee et al. (1993) or Ray and Vohra (1999) model merger processes through a kind of sequential game. However, this approach suffers from one major drawback: The formation of mergers and the distribution of profit typically depend on the order of offers and counter-offers.

The third approach, which we also adopt in our paper, applies a cooperative bargaining game. We analyze a rather general asymmetric oligopoly model developed by Horn and Persson (2001a) who describe possible coalition outcomes by a partition function proposed by Thrall and Lucas (1963). A binary dominance relation ranks coalition outcomes, which reflect ownership structures in our case. Thereby, we can predict equilibrium ownership structures, using the solution concept of the core.

Horn and Persson (2001b) use this approach to analyze a symmetric international oligopoly. The approach has also been applied by Huck and Konrad (2004) and Lommerud et al. (2006) to analyze international mergers between initially symmetric firms. As in our paper, Stadler and Neubecker (2009) apply the cooperative bargaining approach to analyze horizontal merger formation in oligopolies where firms are heterogenous in production technologies. In their model, firms differ in production cost, but, in contrast to our paper, Stadler and Neubecker do not allow for firms to innovate. They show that more heterogenous firms (with large cost asymmetries) merge if cost differences across the industry are substantial, while rather similar

firms (with small cost asymmetries) merge if cost differences across the industry are negligible.

Finally, in order to model process innovation we use the quadratic R&D expenditure function proposed by D'Aspremont and Jacquemin (1988), assuming that R&D investment has diminishing returns.

The remainder of the paper is organized as follows. Section 3.2 describes our model that we use to analyze horizontal merger bargaining between three firms that are heterogenous in innovation technologies. Firms account for the impact on innovation and product market competition when they reach merger decisions. Section 3.3 derives equilibrium market structures and presents results. In section 3.4, a welfare analysis is conducted and policy conclusions and model limitations are discussed. Finally, section 3.5 concludes.

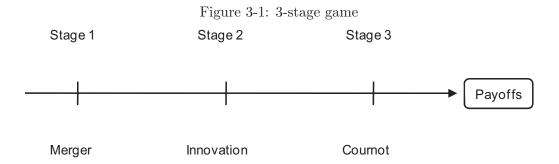
#### 3.2 The Model

Let us consider a market with three firms that supply homogenous products, facing a standard linear demand function. Firms choose output and R&D expenditures which decrease their marginal cost through process innovations. The cost for these innovations differ for the three firms, i.e., firms are heterogenous in their R&D efficiency. Absent any innovation, firms face constant, identical marginal cost. Furthermore, firms are free to negotiate about mergers. In case of a merger, the merged entity uses the most efficient R&D technology available to the previously independent competitors. Not surprisingly, a merger to a monopoly is the preferred option for all firms. As antitrust authorities usually not approve mergers to monopoly, we exclude this option from our set of potential outcomes. Additionally, we do not allow any market structure that squeezes one or more firms out of the market.

The merger game consists of three stages (Figure 1). In stage 1, owners of the three firms negotiate to form coalitions. While the owners of a merged entity are free to negotiate any division of profit between them, payments to third parties are not allowed. The firms that are formed in the first stage invest in innovation in stage 2, i.e., firms invest in reduction of marginal cost. In stage 3, they compete non-cooperatively in Cournot fashion in an oligopolistic market.

#### 3.2.1 The Pre-Merger Market

To keep the model simple, we confine the analysis to the case of n=3 independent firms in the pre-merger market. Firms are assumed to face a linear demand function:



$$p = \max\{A - Q, 0\} \tag{3.1}$$

where Q is the market output. Without innovation, all firms have constant, identical marginal cost (MC):

$$MC = c^0 < A \tag{3.2}$$

For simplicity, we assume that there are no fixed cost.

#### 3.2.2 The Merger Stage

Let us consider a merger as the formation of a coalition between various firms' owners. We express the coalition formation as a partition function form game which can be characterized by a set of outcomes and a binary dominance relation dom defined on the outcomes. The set of outcomes consists of market structures, more precisely, of ownership structures. The binary dominance relation determines the equilibrium post-merger ownership structures. Following Horn and Persson (2001a), let  $M^i$  be a partition of the set of n=3 owners into coalitions. Define M as the set of all possible ownership structures and  $K^i$  the set of all coalitions in  $M^i$ . An ownership structure  $M^j$  dominates  $M^i$  if the owners who are able to impose either  $M^j$  or  $M^i$ , prefer  $M^j$  over  $M^i$ . The owners are considered to bargain over the two structures. These owners are 'decisive' with regard to  $M^j$  and  $M^i$  as they determine the ranking of the two ownership structures. This decision group of owners is defined as follows. Define  $\Re^i$  as the subset of firms (coalitions) in  $M^i$  with  $\Re^i \subseteq K^i$  and define  $O(\Re^i)$  as the set of owners

participating in the firms in  $\Re^i$ . Let  $D^{ij}$  denote the decision group of owners with regard to  $M^i$  and  $M^j$  and the corresponding sets of firms  $\Re^i$  and  $\Re^j$  possessed by these owners in the respective ownership structure. Then  $D^{ij}$  is obtained by:

- (a)  $D^{ij} = O(\Re^i) = O(\Re^j) \neq \emptyset$
- (b)  $\Re^i \cap \Re^j = \emptyset$
- (c)  $\nexists D \subset D^{ij} \mid D$  fulfills (a) and (b)

Thus, firms in  $M^i$  and  $M^j$  with the same set of owners are combined (condition a), but there is no firm that is part of both combinations (condition b), and there is no subset of the formed set that can be formed following the same approach (condition c). For illustration, consider the following example: let  $M^0 = \{1, 2, 3\}$  and  $M^A = \{12, 3\}$  be two ownership structures with no coalition whatsoever in  $M^0$  and a coalition between owner 1 and 2 in  $M^A$ . Owner 3 belongs to the same firm in both structures (violation of condition b). Hence, he cannot influence the ranking of  $M^0$  and  $M^A$ , i.e., owner 3 is not 'decisive' with regard to  $M^0$  and  $M^A$ . However, owners 1 and 2 have the possibility as well as the incentive to influence the ranking of  $M^0$  and  $M^A$ . Thus, the decision group with respect to  $M^0$  and  $M^A$  is  $D^{0A} = \{1,2\}$ . As we confine ourselves to three firms in this paper there is only one decision group with respect to two ownership structures in general.

Following this concept, the binary dominance relation dom can be defined. A market structure  $M^j$  dominates  $M^i$  via  $D^{ij}$  only if the combined profit of the group of owners which is able to enforce these structures is larger in  $M^j$  than in  $M^i$ :

$$\sum_{k \in \Re^j} \pi_k^j > \sum_{k \in \Re^i} \pi_k^i \tag{3.3}$$

Thus, the merging owners are assumed to agree on a profit division making every member of the decision group favor the structure with larger combined profit. Accordingly, there is no payoff division rule for equilibrium mergers defined. Only in the case of no merger the payoff vector is defined as payments between firms are not allowed.

Having defined how to rank any pair of market structure by specifying decision groups and the *dom* relation the solution concept of the core is applied to predict equilibrium ownership structures. The set of equilibrium ownership structures consists of those structures that are in the core:

$$M \setminus \{M^i \in M \mid \exists M \in M \text{ such that } M \text{ dom } M^i\}$$
 (3.4)

Thus, there are only those structures in the core that are not dominated by any market structure. Note that the solution concept of the core does not allow for any structures to be in the core that are only dominated by structures outside the core, i.e., structures that are itself dominated. This seems plausible for a merger as otherwise decision owners could make higher profit by turning to ownership structures that are not in the set of equilibria. Other approaches like the von Neumann-Morgenstern stable set<sup>3</sup> would include these ownership structures in the set of equilibria considering this domination as not credible as it comes through an itself dominated structure.

#### 3.2.3 The Innovation Stage

The firms that are formed in the first stage - including the case of no merger - invest in reduction of marginal cost in the second stage. Using a standard R&D expenditure function by D'Aspremont and Jacquemin (1988) the cost for these process innovations are

$$K(c_k) = \frac{1}{2}k_k(c_k - c^0)^2 \text{ with } 0 \le c_k \le c^0$$
(3.5)

Firms lower their initial marginal cost  $c^0$  to marginal cost  $c_k$  by investing  $K(c_k)$  in process innovation.

Conjecture 3 Firms are different in the efficiency or productivity of their R&D technology, defined by parameter  $k_k$ . The difference in innovation cost between the three firms is fix denoted by  $\Delta > 0$ .  $\Delta = 1$  in this model without loss of generality<sup>4</sup>.

Following conjecture 1 the innovation cost  $k_k$  are for the three firms:

$$k_1 + 2 = k_2 + 1 = k_3 \tag{3.6}$$

 $<sup>^3\</sup>mathrm{See}$ von Neumann, J. and O. Morgenstern (1947)

<sup>&</sup>lt;sup>4</sup> The size of asymmetry, i.e., different values for  $\Delta$ , does not change the general results, but only the specific intervals for  $k_1$  to a small extend that we will present later. There is only one exception for a very small interval of  $k_1$ .

Conjecture 4 Innovation cost  $k_k$  exceed a certain value,  $k_k > \frac{3}{2}$ .

Conjecture 2 guarantees interior solutions. If innovation cost are too low, the most efficient firm can reduce its marginal cost that much that less efficient firms are forced to leave the market.  $k_k > \frac{3}{2}$  ensures no market exit of firm 2 and 3 (in the case of no merger), other market structures and the SOC for a maximum are less restrictive. We derive this barrier in the appendix. The R&D expenditure function is quadratic and R&D investments have diminishing returns. Without innovation firms have constant marginal cost  $c^0$ .

Conjecture 5 Initial marginal cost  $c^0$  exceed a certain value<sup>5</sup>,

$$c_0 \ge \begin{cases} \frac{3}{2} \frac{(2k_1 + 1)(2k_1 - 1)A}{16k_1^3 - 22k_1 + 3} & if \quad k_1 \in ]1.5; 1.876] \\ \frac{4}{3} \frac{(3k_1 + 2)A}{(3k_1 - 2)(3k_1 + 4)} & if \quad k_1 > 1.876 \end{cases}$$

$$(3.7)$$

Conjecture 3 guarantees positive, post-innovation cost  $c^0$ . The lower bound for  $c_0$  depends on the level of innovation cost in the market determined by the innovation cost for firm 1, denoted by  $k_1$ , and the strength of demand, denoted by A. If the pre-innovation cost  $c^0$  relative to the demand A is not too high, then (3.7) implies that the R&D technology, i.e., the innovation cost  $k_1$ , should not be too efficient. If not, the most efficient firm will make very large R&D investments, resulting in zero or even negative post-innovation cost and a squeeze out of other firm(s).

Firms are assumed to maximize profit facing the following profit function:

$$\pi_k = (p - c_k)q_k^* - \frac{1}{2}k_k(c_k - c^0)^2$$
(3.8)

Conjecture 6 Firms are assumed to have no fixed cost for simplicity.

Not surprisingly, a R&D investment, accounted in as fixed cost  $c_k - c^0$  in the profit function, is the more profitable the more units a firm produces.

<sup>&</sup>lt;sup>5</sup> This barrier is most restrictive in the case of no merger  $(k_1 \in ]1.5; 1.876])$  or in the case of a merger between the two most efficient firms  $(k_1 > 1.876)$  and derived in the appendix.

#### 3.2.4 The Production Stage

In the production stage,  $m \leq 3$  firms compete in Cournot fashion producing at different unit cost  $c_k$ , depending on the merger decisions in stage 1 and R&D investment decisions in stage 2. The Cournot-Nash equilibrium is given by

$$\pi_k^* = (q_k^*)^2 = \left(\frac{A - mc_k + \sum_{k' \neq k} c_{k'}}{m+1}\right)^2 \tag{3.9}$$

for any firm k = 1, ..., m in the post-merger market structure.

#### 3.3 Equilibrium Market Structure

In stage 1, n=3 firms bargain about mergers. There are four possible market structures including the pre-merger market structure:

$$M^0 = \{1, 2, 3\}; M^A = \{12, 3\}; M^B = \{13, 2\}; M^C = \{23, 1\}$$
 (3.10)

To derive equilibrium market structures resulting from endogenous merger decisions, the dominance criterion (3.3) has to be applied to all market structures and decision groups. Thus, the profit of all firms have to be derived to calculate profit of the decision groups for the respective market structures then.<sup>6</sup> Table 1 summarizes the results of the comparisons of profit of the decision groups.<sup>7</sup>

Table 1: Equilibrium ownership structures

radio il Equiporati di incressi piete del del			
Level of innovation cost $k_1$	Ownership structures	Innovation cost	
$k_1 \in ]1.5; 1.506]$	$M^0 = \{1, 2, 3\}$	$k_1, k_1 + 1, k_1 + 2$	
$k_1 \in ]1.506; 4.649]$	$M^A = \{12, 3\}$	$k_1, k_1 + 2$	
$k_1 \in ]4.649; \infty[$	$M^0 = \{1, 2, 3\}$	$k_1, k_1 + 1, k_1 + 2$	

Hence, merger formation significantly depends on the innovation cost of the most efficient

<sup>&</sup>lt;sup>6</sup>See appendix for results.

<sup>&</sup>lt;sup>7</sup>As mentioned in section 3.2, the size of asymmetry between firms, i.e., different values for  $\Delta$ , change the specific intervals for  $k_1$  to a small extend only.

firm, which determines the R&D efficiency, i.e., the level of innovation cost, in the industry. The findings are:

- (a) There is no merger involving the least efficient firm 3 for any  $k_1$
- (b) For a very small interval of low  $k_1$  ( $k_1 \in ]1.5; 1.506$ ) there is no merger.

The market structure  $M^0 = \{1, 2, 3\}$  remains unchanged.<sup>8</sup>

- (c) For moderate innovation cost the least efficient firm 3 stays alone and the other two firms merge.  $M^A = \{12, 3\}$  is the corresponding ownership structure.
- (d) For high innovation cost  $(k_1 > 4.649)$  there is no merger whatsoever.

Although it might be expected intuitively that the most and the least efficient firm merge to reach the largest efficiency effect there is no merger involving the least efficient firm. We also want to answer why firms do not merge. Thus, an analysis of the influence of the model's parameters is indicated.

Comparative statics within a market structure. To analyze how model parameters affect equilibrium outcomes, the merger literature usually simply distinguishes between insiders and outsiders. However, as our model is characterized by firm heterogeneity with respect to investment efficiency we consider it more useful to differentiate between the most efficient firm, i.e., the firm with the lowest investment cost, and the other firm  $(M^A, M^B, M^C)$  or firms  $(M^0)$ . The most efficient firm invests more than the other two firms  $(M^0)$  or firm  $(M^A, M^B, M^C)$  and, therefore, produces a higher quantity, resulting in higher profit. The change in profit for increasing innovation cost is different for the most efficient firm and the other firm(s):

$$\frac{d\pi_l}{dk_1} < 0, (3.11)$$

$$\frac{d\pi_f}{dk_1} > 0, (3.12)$$

where  $\pi_l$  denotes the profit of the most efficient firm while  $\pi_f$  denotes the profit of the other firm(s). While the most efficient firm suffers from an increasing innovation cost  $k_1$ , profit of the less efficient firm(s) increase with innovation cost. This is due to the effect of  $k_1$  on

<sup>8</sup> If the asymmetry in R&D efficiency is sufficiently high, (i.e., if  $\Delta \ge 1, 235$ ), this small interval without merger is replaced by a merger between the two most efficient firms, i.e.,  $M^A = \{12, 3\}$ .

innovation activity in the market. The more expensive it is to reduce marginal cost (i.e., the higher  $k_1$ ), the less the most efficient firm invests into cost reductions. The relative advantage of the most efficient firm decreases with an increase in  $k_1$ , and the asymmetry between the firms decreases. In contrast, firm 1 becomes less competitive and produces less, leading (according to the Cournot logic) to an output expansion of firm 2 and 3, which makes cost reductions more profitable for the latter (as they apply to larger quantities). If, however, the innovation cost exceeds a certain threshold of  $k_1^*$  the positive effect of increasing innovation cost  $k_1$  on quantities is outweighed by the negative effect on total innovation cost. Thus, the less efficient firm(s) reduce investments into cost reductions for  $k_1 > k_1^*$ . However, the less efficient firm(s) generally increase the quantity supplied with an increasing innovation cost parameter  $k_1$  due to the reduced quantity supplied by the most efficient firm. Still, total industry output decreases with an increasing innovation cost.

In summary, the behavior of the most efficient firm influences investments into cost reductions and quantities supplied of all firms. Less efficient firms prefer a high level of innovation cost in the industry, as it increases their competitiveness in the innovation market resulting in higher output and higher profit.

It may also be noted that - as expected - firms' profit decrease with an increasing initial marginal cost level  $c^0$ :

$$\frac{d\pi_i}{dc^0} < 0 \tag{3.13}$$

Endogenous merger pattern. Let us now determine how mergers are formed endogenously. Two effects are decisive to determine which mergers are most profitable: The market-power effect and the investment efficiency effect. Firstly, different mergers have different implications for the merging firms' competitiveness vis-à-vis the outsider (market power effect). The difference in firms' productivity levels (after the investment stage) determines output and profit levels. The other important effect to consider are the investment cost savings, as the less efficient firm obtains access to a superior technology through the merger.

Generally, we say that market structure  $M^i$  dominates market structure  $M^j$  if the combined

 $<sup>{}^9</sup>K(c_i) = \frac{1}{2}(k_1 + \Delta)(c_i - c^0)^2$  with  $\Delta = 1 \vee \Delta = 2$  depending on market structure and firm.

profit of the decision group is higher in market structure  $M^i$  than in market structure  $M^j$ .

**Domination of M**<sup>B</sup>= $\{13,2\}$  and **M**<sup>C</sup>= $\{1,23\}$ .  $M^B = \{13,2\}$  and  $M^C = \{1,23\}$  are dominated by market structure  $M^A = \{12,3\}$  for all  $k_1$  and not in the set of equilibrium structures, applying the concept of the core. Defining the binary dominance relation between these three structures, all three players are part of the decision group:  $D^{AB} = D^{AC} = D^{BC} =$ {1,2,3}. Thus, industry profit is compared to obtain equilibrium market structures. Industry profit is highest in the market structure in which the least efficient firm is not part of any merger  $(M^A = \{12, 3\})$ , i.e., the industry innovation cost level is the highest. The efficiency difference between the merging firms is lower in market structure  $M^A$  than in  $M^B$  and  $M^C$ , but competition in the innovation market is also softer under  $M^A$ . In market structure  $M^A$  the merged firm prefers a competitor with low innovation efficiency resulting in lower investment levels and lower quantity supplied by that competitor. Hence, in market structure  $M^A$  a higher proportion of overall quantity is supplied by the more efficient firm than in market structure  $M^B$  and  $M^C$ . The decision group  $D^{AB} = D^{AC} = D^{BC} = \{1, 2, 3\}$  consists of the owners of all three firms; hence, the group maximizes industry profit. A high proportion of overall quantity supplied by the more efficient firm (as in  $M^A$  compared to  $M^B$  and  $M^C$ ) has a positive impact on industry profit. Additionally, the overall quantity supplied in the industry is slightly lower in  $M^A$  than in  $M^B$  and  $M^C$ , also resulting in higher industry profit.

**Domination of M**<sup>0</sup>= $\{1,2,3\}$  and M<sup>A</sup>= $\{12,3\}$ . Although not being dominated by  $M^B$  and  $M^C$  for any  $k_1$ ,  $M^A = \{12,3\}$  is dominated by  $M^0 = \{1,2,3\}$ . For some  $k_1$  the set of equilibrium market structures consists of  $M^A$  and for some  $k_1$  it consists of  $M^0$ .

For a very small interval of (low) innovation cost parameter  $k_1$ , i.e.,  $k_1 \in ]1.5; 1.5063]$ , there is no merger. The two less efficient firms hardly produce anything due to their very limited competitiveness.<sup>10</sup> The efficiency leader almost holds a monopoly. Thus, the most efficient firm does not face strong incentives to merge with any firm, as the market-power effect is very small. For a range of small to moderate innovation cost levels  $(k_1 \in ]1.5063; 4.6488]$ ), the two most efficient firms merge  $(M^A = \{12, 3\})$ . Starting from the pre-merger scenario  $M^0$ , the two less efficient firms significantly increase output with increasing  $k_1$  forcing the most efficient firm to

<sup>&</sup>lt;sup>10</sup>See appendix for details on individual output.

decrease investment into cost reducing innovations and, therefore, the quantity produced. On the revenue side, the market-power effect is the main driving force for a merger between the two most efficient firms ( $M^A = \{12,3\}$ ), as they significantly contract output, resulting in higher prices and, accordingly, profit. The investment efficiency effect renders the merger profitable. The less efficient firm obtains access to the superior technology of the merging partner. Even more important is the low competitiveness of the least efficient firm in the innovation market, resulting in low investment and low output in the pre-merger market.<sup>11</sup> The least efficient firm, which is not involved in the merger, benefits from the output contraction and increases its output, resulting in higher profit. However, the firm is still a relatively "weak" competitor. The newly formed entity only invests more into cost reducing innovations under market structure  $M^A$  than the efficiency leader in the no-merger scenario  $M^0$  if the innovation cost level exceeds a certain threshold.<sup>12</sup> For a high level of investment efficiency in the industry, i.e., a low innovation cost parameter  $k_1$ , the merged firm 12 under scenario  $M^A = \{12,3\}$  invests less than the efficiency leader firm 1 in  $M^0$ , and it produces even less after merging with firm 2 in  $M^A$  than as a separate firm in  $M^0$ .

The last result may be somewhat surprising. While it is well known that a merged entity produces less than the two formerly independent firms together, it may be surprising that a merged entity produces less than only one of the firms in the pre-merger market. The reason is the low competitiveness of the outsider firm 3 in  $M^A$ , as low innovation cost  $k_1$  result in a high market-power effect of the merger. Firm 1 faces two inefficient firms characterized by limited competitiveness in the pre-merger market for a low innovation cost level. After merging with firm 2 it faces only one competitor, which is even the least efficient firm in the market. Thus, the merged entity almost holds a monopoly in the case of a low innovation cost level, resulting in a lower incentive to innovate. Due to this market position firm 1 produces even less after merging with firm 2 in  $M^A$  than as a separate firm in  $M^0$ . Thus, the innovation cost is spread over a lower quantity, making investments less profitable. In contrast, if the innovation cost level  $k_1$  exceeds a certain threshold the merged firm 12 produces more than firm 1 as a separate firm in  $M^0$ , making increased investments profitable.

<sup>&</sup>lt;sup>11</sup>For this reason the most efficient firm does not merge with the least efficient firm.

<sup>&</sup>lt;sup>12</sup>See appendix for details on investment comparison.

For high innovation cost levels  $(k_1 \in ]4.6488; \infty[)$  there is no merger. The two most efficient firms would still significantly reduce output following the merger, but the effect on profit would be negative when compared to the no-merger scenario. The standard merger paradox results. As investment into cost reducing innovations is rather costly now, the equilibrium investment level is low for all firms. Thus, the least efficient firm is relatively competitive in the product market. The least efficient firm would benefit from merger between the two most efficient firms, as in the classical case analyzed by Salant et al. (1983).

In summary, the two most efficient firms merge for a low to moderate innovation cost level, while the least efficient firm remains independent. Although the efficiency gains would be higher if the most efficient and least efficient firms were to merge, the efficiency leader does not merge with the least efficient firm, as the least efficient firm is also a weak competitor. For a high innovation cost level there is no merger whatsoever. Efficiency gains are relatively unimportant in this case, as investment levels are rather low anyhow.

Comparing the results to the literature. Our results demonstrate that the merger paradox of Salant et al. (1983) can be overcome once we introduce heterogeneity in the firms' R&D efficiency. While we do not explicitly account for any merger specific synergies, resulting from a merger, closing down the less efficient R&D department of the two merging firms may be interpreted as a merger-related synergy. For high levels of R&D efficiency our model also yields different results than Kleer (2006) and Vives (2008) who show that, in symmetric market situations, a reduction in the number of firms tends to increase R&D effort. In contrast, we show that R&D levels may actually decrease following a merger, due to the asymmetry in our model.

There are a few theoretical papers that endogenize the merger decision and account for asymmetries between firms in some way, but there is no paper that also allows firms to innovate. Stadler and Neubecker (2009) use the same approach applied in our model to endogenize merger formation and assume firms to be asymmetric in production cost. In their model, merger formation depends on the degree of asymmetry between firms in their production cost levels. In our model, where firms are able to innovate with differences in their R&D efficiency, the merger decisions are not affected by the size of asymmetry between firms<sup>13</sup>, but the overall

<sup>&</sup>lt;sup>13</sup> As already mentioned, the size of the asymmetry only affects the specific threshold values for the innovation

level of R&D efficiency in the industry. Kamien and Zang (1990, 1991) analyze a one shot game where homogenous firms simultaneously bid for other firms and ask prices for the own firm. They find that in a market with three firms facing linear demand there is no merger whatsoever as firms expect to make larger profit by becoming a duopolist competing with the other two firms. Replacing our cooperative bargaining game by the simultaneous game applied by Kamien and Zang (1990, 1991) without changing the innovation stage and still accounting for asymmetry in innovative ability we obtain the following results. The simultaneous game predicts the same market structure as our cooperative bargaining game for levels of R&D efficiency that result in no change in market structure ( $M^0$ ). A merger between the two most efficient firms ( $M^A$ ) is a subgame perfect Nash equilibrium (SPNE) in the simultaneous game for the same level of R&D efficiency, too. In contrast to our cooperative bargaining game, the simultaneous game provides a second SPNE for this efficiency level, a merger between the most efficient firm and the least efficient firm ( $M^B$ ).

#### 3.4 Welfare and policy conclusions

Total welfare  $W^i$  is defined by the sum of consumer surplus  $(CS^i)$  and firms' profit:

$$W^i = CS^i + \sum_k \pi_k^i \tag{3.14}$$

Consumer surplus is given as:

$$CS^{i} = \frac{(Am - \sum_{k} c_{k})^{2}}{2(m+1)^{2}}$$
(3.15)

The welfare effects of mergers can be assessed by comparing welfare for all equilibrium ownership structures predicted in section 3.3.<sup>14</sup> Table 2 summarizes the results of the comparisons of welfare levels for the respective equilibrium market structures. It also compares the welfare maximizing ownership structure with the equilibrium ownership structure predicted by the model:

cost parameter  $k_1$  but not the general results.

<sup>&</sup>lt;sup>14</sup>See appendix for details.

Table 2: Welfare maximizing ownership structures

Level of innovation	Equilibrium ownership	Welfare maximum
$ \cos k_1 $	structures	
$k_1 \in ]1.5; 1.506]$	$M^0 = \{1, 2, 3\}$	$M^0$
$k_1 \in ]1.506; 4.649]$	$M^A = \{12, 3\}$	$M^0(k_1 \in ]1.506; 1.612])$
		$M^A(k_1 \in ]1.612; 1.926])$
		$M^{B \lor C}(k_1 \in ]1.926; 3.170])$
		$M^0(k_1 \in ]3.170; 4.649])$
$k_1 \in ]4.649; \infty[$	$M^0 = \{1, 2, 3\}$	$M^0$

In line with equilibrium ownership structures, welfare maximizing ownership structures significantly depend on the R&D efficiency in the industry. Market outcomes without a merger are socially desirable. In cases of a merger  $(M^A)$  welfare is increased for a wide range of differences in innovation efficiency  $k_1$  but also decreased for a significant range of  $k_1$ . For a range of  $k_1$  mergers involving the least efficient firm  $(M^B \text{ or } M^C)$  would be even superior to the predicted market outcome - a merger between the most efficient firms  $(M^A)$  - from a social point of view. Consumers generally are harmed by merger as output is reduced. In cases of a positive impact of a merger on welfare the negative effect on consumers is dominated by the positive effect on industry profit that results from market power and investment efficiency effects of a merger. In summary, the predicted equilibrium ownership structures are socially desirable for a wide range of R&D efficiency levels in the industry. Only for small range of R&D efficiency levels a no-merger scenario would be preferred from a social point of view although a merger is predicted.

Regarding state aid granted to support innovation, our model suggests that R&D subsidies are not necessarily welfare enhancing once we account for endogenous merger formation. If we assume an innovation cost level of  $k_1 = 5$  and a 10% subsidy on innovation cost so that  $k_1$  is reduced to  $k_1 = 4.5$  from the firms' perspective a merger between firms 1 and 2 will be triggered even though the initial equilibrium market structure is welfare maximizing. However, due to the subsidy,  $M^A = \{12,3\}$  is predicted, i.e., the two most efficient firms merge. A welfare maximizing regulator would still prefer a no-merger scenario. In contrast, if the innovation

subsidy does not influence the endogenous merger formation, the welfare impact of innovation subsidies is generally positive (neglecting the shadow cost of taxation), as an increase in R&D efficiency, i.e., an decrease in  $k_1$ , has a positive impact on social surplus as long as the market structure is not affected.

#### 3.5 Conclusion

This paper has analyzed endogenous horizontal merger formation in a market with three players where firms engage in process innovations. Firms differ in their investment efficiency for cost reducing innovations. In our 3-firm model we allow owners of the three firms to bargain over mergers in the first stage. The merger formation is the outcome of cooperative decisions, and the solution concept allows for any division of profit between merger partners. The asymmetry in R&D efficiency significantly influences the merger decisions. In stage 2 the firms that have been formed in the first stage invest into innovation, i.e., the remaining entities invest into a reduction of their marginal cost level, before they compete in quantities in stage 3. The key results obtained are the following:

Merger formation depends on the R&D efficiency in an industry, i.e., the level of innovation cost. There is no merger outcome involving the least efficient firm. For a low to moderate level of innovation cost the two most efficient firms merge, while the least efficient firm remains independent. For a high level of innovation cost there is no merger whatsoever. Market outcomes without a merger are socially desirable. Welfare is either increased or decreased in the predicted merger case. Accounting for endogenous merger formation R&D subsidies are not necessarily positive depending on the R&D efficiency in the industry, but R&D subsidies are positive if they do not influence merger decisions.

Admittedly, our model is not without limitations. Firstly, we only allow for three firms in order to limit the computational complexity of our model. An increase in the number of firms would diminish the market-power effect of a merger, thereby decreasing the incentives to merge. Applying the solution concept of the core in the context of our cooperative bargaining approach, more than two firms could merge or more than one merger could be predicted in the post-merger ownership structure. Secondly, our approach does not give information about the division of profit within the merged entities. This could be addressed by adding a payoff

division rule that is applied to equilibrium mergers. Profit could be divided equally or according to Shapley values. However, as Horn and Persson (2001a) note there is no core payoff vector as there are incentives to deviate from a proposed payoff vector. Finally, we do not allow for the entry of new competitors. Focusing on product innovation instead of process innovation may be another field of interest but would change the focus of our model to the demand side.

#### 3.A Appendix

#### **3.A.1** Lower bound for $k_1$

Market exit. If innovation cost are too low, the most efficient firm can reduce its marginal cost that much that less efficient firms are forced to leave the market. We derive the value of innovation cost  $k_1$  for the less efficient firm(s) where the optimal marginal cost  $c_k^*$  equals the initial marginal cost  $c_k^0$ , i.e., the cost where a firm does not innovate whatsoever. After calculating this lower bound for the less efficient firm(s) for all market structures the most restrictive value, i.e., the highest  $k_1$ , determines the lower bound for  $k_1$  assuring no market exit of any firm in any possible market structure.

$$M^0 = \{1, 2, 3\}$$
:

$$c_3^* = \frac{18c_0 - 9A + 24Ak_1 - 68c_0k_1 - 12Ak_1^2 + 32c_0k_1^3}{32k_1^3 - 12k_1^2 - 44k_1 + 9} = c^0$$
(3.16)

$$\Leftrightarrow k_1 = \left\{ \frac{3}{2}, \frac{3}{2} - \Delta \right\} \tag{3.17}$$

$$c_2^* = \frac{9A + 12Ak_1 - 56c_0k_1 - 12Ak_1^2 + 32c_0k_1^3}{32k_1^3 - 12k_1^2 - 44k_1 + 9} = c^0$$
(3.18)

$$\Leftrightarrow k_1 = \left\{ \frac{3}{2}, \frac{3}{2} - 2\Delta \right\} \tag{3.19}$$

If  $k_1 > \frac{3}{2}$ , firm 2 and 3 cannot be pushed out of the market.

$$M^A = \{12, 3\}$$
:

$$c_3^* = \frac{16A - 48c_0 - 12Ak_1 + 18c_0k_1 + 27c_0k_1^2}{6k_1 + 27k_1^2 - 32} = c^0$$
(3.20)

$$\Leftrightarrow k_1 = \frac{4}{3} \tag{3.21}$$

If  $k_1 > \frac{4}{3}$ , firm 3 cannot be pushed out of the market.

$$M^B = \{13, 2\} \wedge M^C = \{1, 23\}$$
:

$$c_2^*(M^B) = c_{23}^*(M^C) = \frac{16A - 24c_0 - 12Ak_1 - 9c_0k_1 + 27c_0k_1^2}{27k_1^2 - 21k_1 - 8} = c^0$$
(3.22)

$$\Leftrightarrow k_1 = \frac{4}{3} \tag{3.23}$$

If  $k_1 > \frac{4}{3}$ , firm 2  $(M^B)$  or firm 23  $(M^C)$  cannot be pushed out of the market.

Market structure  $M^0$  determines the lower bound for  $k_1$  with  $k_1 > \frac{3}{2}$  assuring no market exit of firm 2 and 3.

**Second order condition.** The second order condition (SOC) assures concavity of the profit function. Thus,  $k_k$  must satisfy the following conditions.

$$M^0 = \{1, 2, 3\}$$
:

$$\frac{d^2\pi_k}{dc_k^2} = \frac{9}{8} - k_k < 0 {3.24}$$

$$\Leftrightarrow k_k > \frac{9}{8} \tag{3.25}$$

$$M^A = \{12,3\} \wedge M^B = \{13,2\} \wedge M^C = \{1,23\}:$$

$$\frac{d^2\pi_k}{dc_k^2} = \frac{8}{9} - k_k < 0 {3.26}$$

$$\Leftrightarrow k_k > \frac{8}{9} \tag{3.27}$$

SOC is satisfied in all cases as  $k_1 > \frac{3}{2}$  is assumed.

#### **3.A.2** Lower bound for $c_0$

To guarantee positive, post-innovation cost,  $c^0$  has to be higher than a certain bound. The bound for the most efficient firm is critical in each market structure as the most efficient firm can innovate at the lowest cost resulting in the highest investments.

$$M^0 = \{1, 2, 3\}$$
:

$$c_1^* = \frac{3A + 6c_0 - 44c_0k_1 - 12Ak_1^2 + 32c_0k_1^3}{32k_1^3 - 12k_1^2 - 44k_1 + 9} \ge 0$$
(3.28)

$$\Leftrightarrow c_0 \ge \frac{3}{2} \frac{(2k_1 + 1)(2k_1 - 1)A}{16k_1^3 - 22k_1 + 3} \tag{3.29}$$

$$M^A = \{12, 3\}$$
:

$$c_{12}^* = \frac{18c_0k_1 - 24c_0 - 12Ak_1 - 8A + 27c_0k_1^2}{6k_1 + 27k_1^2 - 32} \ge 0$$
(3.30)

$$\Leftrightarrow c_0 \ge \frac{4}{3} \frac{(3k_1 + 2) A}{(3k_1 - 2)(3k_1 + 4)} \tag{3.31}$$

$$M^B = \{13, 2\} \wedge M^C = \{1, 23\}$$
:

$$c_{13}^*(M^B) = c_1^*(M^C) = \frac{4A - 12c_0 - 12Ak_1 - 9c_0k_1 + 27c_0k_1^2}{27k_1^2 - 21k_1 - 8}$$
(3.32)

$$c_0 \ge \frac{4}{3} \frac{(3k_1 - 1)A}{9k_1^2 - 3k_1 - 4} \tag{3.33}$$

The critical bounds are in  $M^0$  and  $M^A$  depending on  $k_1$  resulting in the following condition for  $c^0$ :

$$c_0 \ge \begin{cases} \frac{3}{2} \frac{(2k_1 + 1)(2k_1 - 1)A}{16k_1^3 - 22k_1 + 3} & \text{if} \quad k_1 \in ]1.5; 1.876] \\ \frac{4}{3} \frac{(3k_1 + 2)A}{(3k_1 - 2)(3k_1 + 4)} & \text{if} \quad k_1 > 1.876 \end{cases}$$

$$(3.34)$$

#### 3.A.3 Binary dominance relations

To derive the binary dominance relations between all market structures we have to calculate the profit of the decision groups for the respective market structures. Thus, we need to calculate profit of all firms in each market structure first.

#### Profit of firms.

$$M^{0} = \{1, 2, 3\}$$

$$\pi_{1}^{0} = \frac{1}{2} \frac{(8k_{1} - 9)(2k_{1} + 1)^{2}(2k_{1} - 1)^{2}(A - c_{0})^{2}k_{1}}{(32k_{1}^{3} - 12k_{1}^{2} - 44k_{1} + 9)^{2}}$$
(3.35)

$$\pi_2^0 = \frac{1}{2} \frac{(8k_1 - 1)(2k_1 - 3)^2 (2k_1 + 1)^2 (A - c_0)^2 (k_1 + 1)}{(32k_1^3 - 12k_1^2 - 44k_1 + 9)^2}$$
(3.36)

$$\pi_3^0 = \frac{1}{2} \frac{(8k_1 + 7)(2k_1 - 3)^2(2k_1 - 1)^2(A - c_0)^2(k_1 + 2)}{(32k_1^3 - 12k_1^2 - 44k_1 + 9)^2}$$
(3.37)

$$M^{A} = \{12, 3\}$$

$$\pi_{12}^{A} = \frac{(9k_{1} - 8)(3k_{1} + 2)^{2}(A - c_{0})^{2}k_{1}}{(6k_{1} + 27k_{1}^{2} - 32)^{2}}$$
(3.38)

$$\pi_3^A = \frac{(9k_1 + 10)(3k_1 - 4)^2(A - c_0)^2(k_1 + 2)}{(6k_1 + 27k_1^2 - 32)^2}$$
(3.39)

 $M^B = \{13, 2\} \wedge M^C = \{1, 23\}$ 

$$\pi_{13}^{B} = \pi_{1}^{C} = \frac{(9k_{1} - 8)(3k_{1} - 1)^{2}(A - c_{0})^{2}k_{1}}{(27k_{1}^{2} - 21k_{1} - 8)^{2}}$$
(3.40)

$$\pi_2^B = \pi_{23}^C = \frac{(9k_1 + 1)(3k_1 - 4)^2(A - c_0)^2(k_1 + 1)}{(27k_1^2 - 21k_1 - 8)^2}$$
(3.41)

**Profit of decision groups and dominance relations.** We then get the profit of the decision groups to apply the dominance criterion to the comparison of two market structures.

 $M^0$  vs.  $M^A$ :

$$M^{0} : D_{12}^{0A} = \frac{1}{2} \frac{(A - c_{0})^{2} (2k_{1} + 1)^{2} (66k_{1} + 28k_{1}^{2} - 136k_{1}^{3} + 64k_{1}^{4} - 9)}{(32k_{1}^{3} - 12k_{1}^{2} - 44k_{1} + 9)^{2}}$$
(3.42)

$$M^{A} : D_{12}^{0A} = \frac{(9k_{1} - 8)(3k_{1} + 2)^{2}(A - c_{0})^{2}k_{1}}{(6k_{1} + 27k_{1}^{2} - 32)^{2}}$$
(3.43)

For the dominance relation we get:

 $M^0 dom \ M^A \text{ for } k_1 \in ]1.5; 1.506] \land [4.632; \infty[,$ 

 $M^A dom \ M^0 \ \text{for} \ k_1 \in ]1.506; 4.632].$ 

 $M^0$  vs.  $M^B$ :

$$M^{0} : D_{13}^{0B} = \frac{(A - c_{0})^{2} (2k_{1} - 1)^{2} (15k_{1} - 88k_{1}^{2} - 4k_{1}^{3} + 32k_{1}^{4} + 63)}{(32k_{1}^{3} - 12k_{1}^{2} - 44k_{1} + 9)^{2}}$$
(3.44)

$$M^{B} : D_{13}^{0B} = \frac{(9k_{1} - 8)(3k_{1} - 1)^{2}(A - c_{0})^{2}k_{1}}{(27k_{1}^{2} - 21k_{1} - 8)^{2}}$$
(3.45)

For the dominance relation we get:

 $M^0 dom \ M^B \text{ for } k_1 \in ]1.5; 1.540] \land ]4.649; \infty[,$ 

 $M^B dom \ M^0 \text{ for } k_1 \in ]1.540; 4.649].$ 

 $M^0$  vs.  $M^C$ :

$$M^{0} : D_{23}^{0C} = \frac{1}{2} \frac{(A - c_{0})^{2} (2k_{1} - 3)^{2} (4k_{1}^{2} - 30k_{1} + 120k_{1}^{3} + 64k_{1}^{4} + 13)}{(32k_{1}^{3} - 12k_{1}^{2} - 44k_{1} + 9)^{2}}$$
(3.46)

$$M^{C} : D_{23}^{0C} = \frac{(9k_{1}+1)(3k_{1}-4)^{2}(A-c_{0})^{2}(k_{1}+1)}{(27k_{1}^{2}-21k_{1}-8)^{2}}$$
(3.47)

For the dominance relation we get:

 $M^0 dom \ M^C \text{ for } k_1 \in ]1.5; \infty].$ 

Thus,  $M^0$  is not dominated, i.e.,  $M^0$  is in the core, for  $k_1 \in ]1.5; 1.506] \land [4.649; \infty[$ .

 $M^A$  vs.  $M^B \wedge M^A$  vs.  $M^C$ :

$$M^{A} : D_{123}^{AB} = D_{123}^{AC} = \frac{2(A - c_0)^2 \left(36k_1^3 - 204k_1^2 - 32k_1 + 81k_1^4 + 160\right)}{\left(6k_1 + 27k_1^2 - 32\right)^2}$$
(3.48)

$$M^{B} \wedge M^{C}$$
:  $D_{123}^{AB} = D_{123}^{AC} = \frac{2(A - c_0)^2 \left(64k_1 - 15k_1^2 - 126k_1^3 + 81k_1^4 + 8\right)}{\left(27k_1^2 - 21k_1 - 8\right)^2}$  (3.49)

For the dominance relation we get:

 $M^A dom \ M^B \wedge M^A dom \ M^C \text{ for } k_1 \in ]1.5; \infty[.$ 

Thus, we get for the equilibrium market structures the following:  $M^0$  is not dominated for  $k_1 \in ]1.5; 1.506] \land ]4.649; \infty[$ ,  $M^A$  is not dominated for  $k_1 \in ]1.506; 4.649]$ .  $M^B$  and  $M^C$  are not in the core for any  $k_1$ .

#### 3.A.4 Investment comparison and individual output

The optimal marginal cost for equilibrium market structures  $M^0$  and  $M^A$  are:

 $M^{0}:$ 

$$c_1^* = \frac{3A + 6c_0 - 44c_0k_1 - 12Ak_1^2 + 32c_0k_1^3}{32k_1^3 - 12k_1^2 - 44k_1 + 9}$$
(3.50)

$$c_2^* = \frac{9A + 12Ak_1 - 56c_0k_1 - 12Ak_1^2 + 32c_0k_1^3}{32k_1^3 - 12k_1^2 - 44k_1 + 9}$$
(3.51)

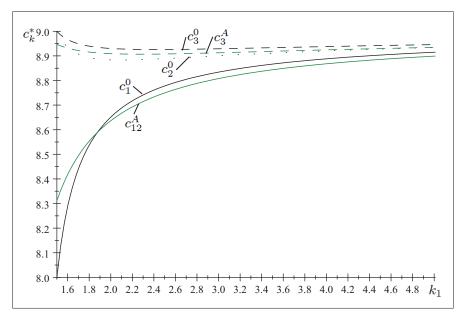
$$c_3^* = \frac{18c_0 - 9A + 24Ak_1 - 68c_0k_1 - 12Ak_1^2 + 32c_0k_1^3}{32k_1^3 - 12k_1^2 - 44k_1 + 9}$$
(3.52)

 $M^A$ :

$$c_{12}^* = \frac{18c_0k_1 - 24c_0 - 12Ak_1 - 8A + 27c_0k_1^2}{6k_1 + 27k_1^2 - 32}$$
(3.53)

$$c_3^* = \frac{16A - 48c_0 - 12Ak_1 + 18c_0k_1 + 27c_0k_1^2}{6k_1 + 27k_1^2 - 32}$$
(3.54)

The next figure plots the optimal marginal cost in market structure  $M^0$  (black lines) and  $M^A$  (green lines) against innovation cost  $k_1$  for A = 10 and  $c^0 = 9$ :



Optimal marginal cost in  $M^0$  and  $M^A$ 

Firm 3 slightly decreases marginal cost, i.e., increases innovation investments, in case of a merger of the other two firms  $(M^A)$ . Firm 1 increases innovation investments after merging with firm 2 only if innovation cost exceed a specific value.

Lower marginal cost result in higher optimal output. The optimal individual output for equilibrium market structures  $M^0$  and  $M^A$  are:

 $M^{\mathbf{0}}:$ 

$$q_1^* = \frac{2k_1(2k_1+1)(2k_1-1)(A-c_0)}{32k_1^3 - 12k_1^2 - 44k_1 + 9}$$
(3.55)

$$q_2^* = \frac{2(2k_1 - 3)(2k_1 + 1)(A - c_0)(k_1 + 1)}{32k_1^3 - 12k_1^2 - 44k_1 + 9}$$
(3.56)

$$q_3^* = \frac{2(2k_1 - 3)(2k_1 - 1)(A - c_0)(k_1 + 2)}{32k_1^3 - 12k_1^2 - 44k_1 + 9}$$
(3.57)

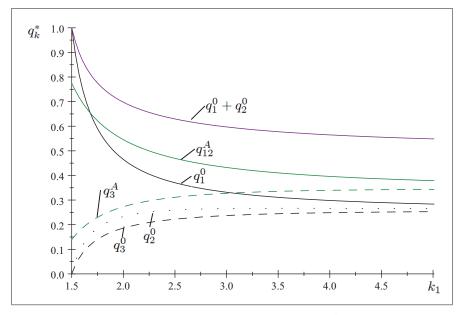
$$q_1^* + q_2^* = \frac{-2(A - c_0)(8k_1 - 8k_1^3 + 3)}{32k_1^3 - 12k_1^2 - 44k_1 + 9}$$
(3.58)

 $M^A$ :

$$q_{12}^* = \frac{3k_1(3k_1+2)(A-c_0)}{6k_1+27k_1^2-32}$$
(3.59)

$$q_3^* = \frac{3(3k_1 - 4)(A - c_0)(k_1 + 2)}{6k_1 + 27k_1^2 - 32}$$
(3.60)

The next figure plots the individual output in market structure  $M^0$  (black lines and purple line for  $q_1 + q_2$ ) and  $M^A$  (green lines) against innovation cost  $k_1$  for A = 10 and  $c^0 = 9$ :



Individual output in  $M^0$  and  $M^A$ 

The merged entity contracts output in comparison to their combined output in  $M^0$  for any  $k_1$ . For low  $k_1$ , firm 1 produces even less after merging with firm 2 in  $M^A$  than as a separate firm in  $M^0$ .

#### 3.A.5 Welfare

Welfare  $W^i$  is defined by the sum of consumer surplus  $(CS^i)$  and firms' profit  $(PS^i)$ :

$$W^{i} = CS^{i} + PS^{i} = \frac{(Am - \sum_{k} c_{k})^{2}}{2(m+1)^{2}} + \sum_{k} \pi_{k}^{i}$$
(3.61)

$$W^{0} = \frac{3}{2} \frac{(A - c_{0})^{2} \left(740k_{1}^{2} - 313k_{1} + 424k_{1}^{3} - 976k_{1}^{4} - 144k_{1}^{5} + 320k_{1}^{6} + 51\right)}{\left(32k_{1}^{3} - 12k_{1}^{2} - 44k_{1} + 9\right)^{2}}$$
(3.62)

$$W^{A} = \frac{4(A - c_0)^2 (72k_1^3 - 192k_1^2 - 88k_1 + 81k_1^4 + 152)}{(6k_1 + 27k_1^2 - 32)^2}$$
(3.63)

$$W^{B} = W^{C} = \frac{4(A - c_{0})^{2} \left(50k_{1} - 57k_{1}^{2} - 90k_{1}^{3} + 81k_{1}^{4} + 22\right)}{\left(27k_{1}^{2} - 21k_{1} - 8\right)^{2}}$$
(3.64)

Comparing welfare levels of all market structures to derive welfare maximizing market structures we get:

$$W^0 > W^A \text{ for } k_1 \in ]1.5; 1.612] \land k_1 > 2.775$$
  
 $W^0 > W^B \land W^C \text{ for } k_1 \in ]1.5; 1.644] \land k_1 > 3.170$   
 $W^A > W^B \land W^C \text{ for } k_1 < 1.926.$ 

## Chapter 4

# Horizontal Divestitures and R&D Incentives in Asymmetric Duopoly

#### 4.1 Introduction

One of the most controversial reform proposals of Germany's competition law has been the suggestion by the Minister of Economics to introduce divestiture powers for the Federal Cartel Office into the Law against Restraints of Competition. While divestiture powers are, by and large, uncontroversial in cases where a dominant firm has (repeatedly) abused its position, the current proposal suggests to include more general divestiture powers even in cases where no abuse has been proven. Proponents of the reform proposal argue that divestiture measures can pro-actively facilitate more intense competition, especially in cases where an abuse is notoriously difficult to prove (see, e.g., Monopolkommission, 2010). In contrast, opponents argue that a divestiture threat is likely to stifle investment and, therefore, to reduce welfare (see, e.g., Satzky, 2010). In fact, there are only a limited number of countries where divestiture is feasible without a proven abuse of dominance, the United Kingdom being the most prominent exception. This paper aims at shedding some light on the question how a threat of (horizontal) divestiture can affect competition and investment. In order to capture the idea of market dominance we will consider an asymmetric duopoly. We also provide for the possibility to invest into marginal cost reducing R&D in order to capture the idea that a divestiture threat affects investment incentives.

While there is a broad industrial economics and management literature on mergers and merger policy, there are hardly any papers on divestiture measures - possibly because divestiture measures have been rather exceptional outside network industries such as electricity, gas, or telecommunications. And even in the latter industries divestiture measures have usually (even though not exclusively) concerned vertically related parts of a value chain.

However, there is a broad literature on the relationship between market structure and R&D incentives. While there is no literature that endogenizes R&D investments and potential divestitures, the relationship between market structure and R&D incentives has been debated since the pioneering works of Schumpeter (1943) and Arrow (1962). Broadly speaking, while Schumpeter (1943) has emphasized how the prospect of (temporary) monopoly profits drives innovation, Arrow (1962) has shown that competitive markets usually provide higher investment incentives than monopolistic market structures. Recently, Yi (1999) and Boone (2001) have contributed to this strand of literature and explored how the intensity of competition affects firms' innovation incentives. While Yi (1999) analyzes how the number of firms affects innovation incentives if firms are homogeneous, Boone (2001) develops a framework with asymmetric firms. None of these papers accounts for the fact though that firms may account for possible changes in market structure when deciding about their R&D investment.

More related to our paper is the literature on the impact of mergers on innovation, as mergers can be seen as the opposite to a divestiture measure. The decisive difference, however, is that mergers are endogenous, while the divestiture measures under consideration here are exogenous. Furthermore, the literature about the effects of mergers on innovations is almost exclusively empirical in nature (see Schulz, 2008). As Cassiman et al. (2005) and Schulz (2008) have recently noted in two surveys of this literature, there is a lack of theoretical research that analyzes how mergers affect innovation. Clearly, there is even less literature on the relationship between divestiture and innovation incentives - to the best of our knowledge there is none at all.

Although there is no research that endogenizes R&D investments and potential divestitures, our work can probably be best compared with the recent paper by Ishida et al. (2010) who also consider an asymmetric Cournot model with strategic R&D investment. In contrast to our model, there is only one low-cost firm, but n-1 high-cost firms. Their paper considers a one

period game and investigates the impact of competition on R&D, i.e. a variation in the number of high-cost firms. They find that an increase in the number of high-cost firms may stimulate the low-cost firm's R&D effort and increase its profit. In contrast, the threat of divestiture generally reduces the low-cost firm's R&D incentives in our model.

The rest of the paper now proceeds as follows: Section 4.2 presents our model and derives main results. In section 4.3, we analyze impact on welfare and discuss implications for competition policy. Finally, section 4.4 concludes and points out topics of interest for further research.

#### 4.2 The Model

#### 4.2.1 Setup

Let us consider the 2-stage game illustrated in Figure 1:

Figure 4-1: 2-stage game

Potential divestiture

Stage 1 Stage 2

Payoffs

Innovation Cournot

The game starts with two heterogeneous firms, a low-cost firm L and high-cost firm H, who face a different (constant) marginal cost of production with  $c_L < c_H$ . In stage 1, the two firms commit to invest into marginal cost reductions, i.e. the two firms are assumed to commit to an irreversible R&D investment. The R&D investment can be described by a function as introduced by D'Aspremont and Jacquemin (1988) with decreasing returns to R&D. Following stage 1, there is an exogenous horizontal divestiture of the low-cost firm with some exogenous probability  $\alpha$  which is known to both firms. Without divestiture the two firms compete in Cournot fashion in the product market. In case of divestiture, the low-cost firm L is split into

two equally efficient, but independent divisions, i.e. an identical sibling of firm L is created, called firm E. Hence, the two former incumbents now compete with a third low-cost entrant (again in Cournot fashion) in stage 2 in case of divestiture.

The innovation stage. The high- and the low-cost firm simultaneously commit to invest into the reduction of their marginal cost in stage 1. Using an R&D function as introduced by D'Aspremont and Jacquemin (1988), the cost functions for the two firms are given by

$$C_i = (c_i - x_i)q_i + x_i^2 (4.1)$$

where  $x_i$  denotes the R&D investment chosen by firm i (with i = L, H) in stage 1 and  $(c_i - x_i)q_i$  is the total cost of production of firm i with  $q_i$  being firm i's output level. The initial marginal cost level is denoted by  $c_i$ , while  $c_i - x_i$  is the marginal cost level after investment. The R&D expenditure function is quadratic  $(x_i^2)$  to ensure diminishing returns of R&D.

The production stage. In stage 2 of the game, firms compete in Cournot fashion in the product market, facing a linear inverse demand function

$$p = \max\{1 - Q, 0\} \tag{4.2}$$

where Q is the total quantity supplied by all firms. Without divestiture, high-cost firm H and low-cost firm L compete, while with divestiture another low-cost firm E is created following stage 1 in order to compete with the two incumbent firms in stage 2. Total output after divestiture is specified as

$$Q^{D} = q_{L}^{D} + q_{H}^{D} + q_{E}^{D} (4.3)$$

where  $q_E$  denotes the output of the entrant firm E.

In order to concentrate the analysis on interior solutions we assume that the initial marginal cost level of the low-cost firm L exceeds a certain threshold  $\underline{c}_L$  (assumption 1) with

$$\underline{c_L} \equiv \frac{1}{12} \frac{(256 + 142\alpha)c_H - 7\alpha^2(1 - c_H) - 10\alpha - 64}{16 + 11\alpha}.$$
(4.4)

Assumption 1 guarantees interior solutions, i.e.  $c_L \ge \underline{c_L}$  is a necessary and sufficient condition for  $x_H \ge 0$ . If the low-cost firm's initial marginal cost of production is lower than  $\underline{c_L}$ ,

the firm would reduce its marginal cost level to such an extent that the high-cost firm would cease production and leave the market. Hence, we focus on non-drastic innovations that allow the inefficient firm to remain in the market. Note that we also abstract from any fixed cost of production other than R&D expenses.

#### 4.2.2 Equilibrium results

Let us now solve the game by backward induction. In stage 2, without divestiture, the Cournot-Nash equilibrium is simply given by

$$q_i^* = q_i^{ND*} = \frac{1 - 2(c_i - x_i) + (c_j - x_j)}{3}$$
(4.5)

for  $i \neq j$  and i, j = L, H. With horizontal divestiture, the firm's profit maximizing output is given as

$$q_L^{D*} = q_E^{D*} = \frac{3}{4} q_L^* \tag{4.6}$$

and

$$q_H^{D*} = \frac{3}{4}q_H^* - \eta \tag{4.7}$$

where  $\eta \equiv ((c_H - x_H) - (c_L - x_L))/4 > 0$ . The entrant obviously produces the same output as the low-cost incumbent because of it being an identical sibling. Optimal output depends on investment levels and ex ante marginal cost of production.

In stage 1, firm L and H simultaneously decide on their R&D investments that reduce marginal cost. When deciding how much to invest, each firm accounts for optimal quantities in stage 2 and the probability of divestiture. The firms choose  $x_i$  so as to maximize their total profit, taking the other firm's choices as given:

$$\max_{x_i} \pi_i(x_i, x_j) = (1 - \alpha)(q_i^*(x_i, x_j))^2 + \alpha(q_i^{D*}(x_i, x_j))^2 - x_i^2$$
(4.8)

The equilibrium investment levels of firm L and H are

$$x_L^* = \tau + \frac{192c_H - 256c_L - 7\alpha^2 c_L - 44\alpha}{224 + 26\alpha - 7\alpha^2}$$
(4.9)

and

$$x_H^* = \tau + \frac{192c_L - 256c_H + 4\alpha c_L - 7\alpha^2 c_H - 58\alpha c_H + 10\alpha}{224 + 26\alpha - 7\alpha^2}$$

$$\tag{4.10}$$

where  $\tau \equiv (64 + 128\alpha c_L - 84\alpha c_H + 7\alpha^2)/(224 + 26\alpha - 7\alpha^2) > 0.1$ 

Optimal outputs and the resulting profits depend on the firms' investment levels. Thus, we obtain the following equilibrium quantities and profits for firm L:

$$q_L^* = q_L^{ND*} = \Theta x_L^* \tag{4.11}$$

$$q_L^{D*} = \frac{3}{4} \Theta x_L^* \tag{4.12}$$

and

$$\pi_L^* = \pi_L^{ND*} = (\Theta^2 - 1)(x_L^*)^2 \tag{4.13}$$

$$\pi_L^{D*} = (\frac{9}{16}\Theta^2 - 1)(x_L^*)^2. \tag{4.14}$$

where  $\Theta \equiv 24/(16-7\alpha) \ge 3/2$ .

For firm H's equilibrium quantities and profits we obtain:

$$q_H^* = q_H^{ND*} = \Lambda x_H^* \tag{4.15}$$

$$q_H^{*D} = \frac{3}{4}\Lambda x_H^* - \eta \tag{4.16}$$

and

$$\pi_H^* = \pi_H^{ND*} = (\Lambda^2 - 1)(x_H^*)^2 \tag{4.17}$$

$$\pi_H^{D*} = (\frac{3}{4}\Lambda x_H^* - \eta)^2 - (x_H^*)^2 \tag{4.18}$$

where  $\Lambda \equiv 6 \frac{16 + 48c_L - 64c_H + 9\alpha c_L - 14\alpha c_H + 5\alpha}{64 + 192c_L - 256c_H + 132\alpha c_L - 142\alpha c_H - 7\alpha^2 c_H + 10\alpha + 7\alpha^2} \ge 3/2$ .

**Proposition 7** For any  $\alpha \in [0;1]$ , individual quantities  $(q_L^{D*}, q_H^{D*})$  and firms' profits  $(\pi_L^{D*}, \pi_H^{D*})$  after divestment are lower than those in the no divestment scenario.

As can be easily seen, each firm's individual output and profit is lower in stage 2 following

Note that the second order conditions are always satisfied:  $d^2\pi_L/dx_L^2 = -(20+7\alpha)/18 < 0$  and  $d^2\pi_H/dx_H^2 = -\left(10-\frac{17}{8}\alpha\right)/9 < 0$  for  $\alpha \in [0,1]$ .

a divestiture, i.e.  $q_i^{D*} < q_i^*$  and  $\pi_i^{D*} < \pi_i^*$  for i = L, H. This is as expected, given the Cournot setting, where rivals' quantities are strategic substitutes. A divestiture measure intensifies market competition so that prices decrease and the two incumbent firms suffer.

Let us now examine how an increase in the probability of divestiture ( $\alpha$ ) affects incumbent firms' investment decisions.

**Proposition 8** For any  $\alpha \in [0; 1]$ , (i)  $x_L^*$  is decreasing in  $\alpha$ ; while (ii) there exists a nonempty interval  $Z^{x_H} \equiv \left[c_L^{x_H}; c_H\right[ \text{ with } c_L^{x_H} > \underline{c_L} \text{ such that } x_H^* \text{ is increasing in } \alpha \text{ if and only if } c_L \in Z^{x_H}.$ 

The proposition has two parts. Firstly, and not surprisingly, firm L's R&D investment always decreases in  $\alpha$ . A threat of divestiture induces firm L to reduce its R&D investment, independent of the severity of the threat (i.e. the level of  $\alpha$ ). As firm L's investment decision also affects the competitiveness of its sibling in case of divestiture, investment incentives unambiguously decrease in  $\alpha$ .

Secondly, and more suprisingly, firm H's R&D investment may be increasing in  $\alpha$ . The key factor is the ex ante difference between  $c_L$  and  $c_H$ . The more likely the emergence of an additional firm is, the stronger are firm H's incentives to invest iff it is sufficiently efficient ex ante compared to firm L. However, this only holds for a certain range of parameter values. The higher  $\alpha$  the smaller is the difference in market share that induces firm H to increase its investment level.

Intuitively one may expect all firms to reduce their R&D investment if the probability of additional competition (due to divestiture) increases as innovation rents decrease. As all firms produce lower quantities, R&D expenses can only be spread over smaller output levels, i.e. divestiture induces scale diseconomies of R&D investment. An increase in  $\alpha$  has a negative impact on the low-cost firms' cost structure due to lower equilibrium R&D. This in turn induces firm H to invest more into R&D, as the rivals' R&D investments are strategic substitutes. This positive effect on firm H's R&D investment only outweighs the negative effect of additional competition if firm H is sufficiently efficient, i.e. if its initial market share (determined by its relative competitiveness) is sufficiently large. Otherwise, the positive effect is not strong enough, and the negative effect of increased competition dominates.

**Proposition 9** For any  $\alpha \in [0;1]$ ,  $q_L^*$  and  $q_L^{D*}$  are decreasing in  $\alpha$ , while  $q_H^*$  and  $q_H^{D*}$  are

increasing in  $\alpha$ .

Since we know from Proposition 8 that  $x_L^*$  is decreasing in  $\alpha$  it is straight forward that  $q_L^*$  and  $q_L^{D*}$  are also decreasing in  $\alpha$ . Maybe somewhat surprisingly,  $q_H^*$  and  $q_H^{D*}$  are increasing in  $\alpha$ . This means that firm H is increasing its output even though its R&D investment is decreasing for a range of parameter values, as we know from proposition 8. This implies that the negative effect of lower R&D on output is more than compensated by the decrease in firm L's quantity which induces firm H's in turn to increase its output.

**Proposition 10** For  $\alpha \in [0; 0.32]$  there exists some nonempty interval  $Z^{\pi_L} \equiv \left[\underline{c_L}; c_L^{\pi_L}\right]$  such that  $\pi_L^*$  is increasing in  $\alpha$  if and only if  $c_L \in Z^{\pi_L}$ . For any  $\alpha \in [0; 1]$ ,  $\pi_H^*$  is increasing in  $\alpha$ .

R&D levels have three effects on the firms' profits: Firstly, there is a direct cost of R&D. Secondly, the R&D reduces the cost per unit of output, and, thirdly, the firms' output changes due to the change in marginal cost. Proposition 10 states that firm H's profit without any divestiture, is generally increasing in  $\alpha$ . Hence, a threat of splitting up the low-cost rival is good news for the high-cost firm - at least provided that no divestiture actually takes place. We know from propositions 8 and 9 that firm H's R&D investment is increasing in  $\alpha$  for a range of parameters and that its output is generally increasing in  $\alpha$ . As a consequence, its variable profit (before R&D expenses) is increasing in  $\alpha$ , and this increase more than compensates firm H's increase in R&D expenditures. Regarding firm L, proposition 10 states that the firm's profit without any divestiture is increasing in  $\alpha$  for low values of  $\alpha$  if the difference in the firms' marginal cost is sufficiently large, i.e. if the low-cost firm is sufficiently dominant. An increase in  $\alpha$  has two main effects. On the one hand, the variable profit decreases, as unit production costs increase due to the reduction in R&D and output decreases (propositions 8 and 9) even though the price in stage 2 increases. On the other hand, R&D expenditures decrease. For low values of  $\alpha$  and large marginal cost differences, lower R&D investments by firm L induce a relatively low increase in its unit production cost so that the decrease in R&D expenditures outweighs the decrease in variable profit in these cases.

**Proposition 11** For any  $\alpha \in [0; 1]$ , (i)  $\pi_L^{D*}$  is increasing in  $\alpha$ ; (ii) there exists some nonempty interval  $Z^{\pi_H^D} \equiv \left[\underline{c_L}; c_L^{\pi_H^D}\right]$  such that  $\pi_H^{D*}$  is decreasing in  $\alpha$  if and only if  $c_L \in Z^{\pi_H^D}$ .

Proposition 11 states how an increase in  $\alpha$  affects post-divestiture profits. While the profit is increasing in  $\alpha$  for the low-cost firms, this is not unambiguously so for the high-cost firm. If the marginal cost difference is relatively large (and firm H, therefore, relatively small) an increase in  $\alpha$  induces lower R&D investment of firm H. An increase in  $\alpha$  now has a positive impact on post-divestiture output  $q_H^{D*}$  (proposition 9) and it also positively affects the firm's profit through a reduction in R&D expenditures. There is also a negative impact on profit as the unit production cost increases. If the difference in marginal cost is relatively large, firm H produces relatively little and also spends relatively little on R&D. A change in its R&D expenditures has a relatively strong impact on its unit production cost due to the diminishing returns of R&D. Accordingly, the profit impact resulting from the increase in its unit production cost outweighs the positive effects in profit.

In summary, we obtain the following key results on the individual firm level. An actual divestiture harms both the high-cost and low-cost firm. A threat of divestiture already reduces firm L's dominance, as it reduces its R&D investment, resulting in a lower competitive advantage. Nevertheless, a threat of divestiture may increase firm L's profit if the high-cost firm is relatively inefficient and firm L, therefore, relatively dominant. With respect to firm H, a threat of divestiture stimulates its R&D investment if the low-cost firm is not too efficient. Additionally, the divestiture threat results in an increase in firm H's profit.

It may be interesting to compare these results with the recent paper by Ishida et al. (2010), who also consider an asymmetric Cournot model with strategic R&D investment. In their model, one low-cost firm competes with n-1 high-cost firms. In contrast to our model they investigate the impact of an increase in the number of high-cost firms. Looking at their model for the duopoly case a marginal increase in n is in some way similar to comparing the divestment with the no divestment scenario in our model (proposition 7). Note that we increase the number of low-cost firms, while they increase the number of high-cost firms. Accordingly, our findings are different. In our model, both low- and high-cost firms are worse off if the divestiture actually takes place. Ishida et al. (2010) find that an increase in n may increase the low-cost firm's profit if it is sufficiently efficient relative to the high-cost firm. Regarding R&D investments they find that an additional high-cost firm induces lower R&D of the high-cost firm, while the effect for the low-cost firm is not unambiguous, depending on its efficiency. In contrast, we find

that the change in the high-cost firm's R&D investment depends on its efficiency, while the low-cost firm generally decreases its R&D expenditures in the divestment scenario.

## 4.3 Welfare and Policy Conclusions

Let us now take a closer look at industry profits  $(\pi)$ , consumer surplus (CS) and total welfare (W). We also investigate the industry-wide rate of innovation by considering total industry investments (I).  $\pi$ , CS, W are defined as

$$CS^* = CS^{ND*} = \frac{1}{2}(Q^*)^2 = \frac{1}{2}(6\frac{32 + \alpha - 16c_L - 16c_H + 13\alpha c_L - 14\alpha c_H}{26\alpha - 7\alpha^2 + 224})^2$$
(4.19)

$$CS^{D*} = \frac{1}{2}(Q^{D*})^2 = \frac{1}{2}(12\frac{18 - 16c_L - 2c_H + 7\alpha c_L - 7\alpha c_H}{26\alpha - 7\alpha^2 + 224})^2$$
(4.20)

$$\pi^* = \pi^{ND*} = \pi_L^* + \pi_H^* \tag{4.21}$$

$$\pi^{D*} = 2\pi_L^{D*} + \pi_H^{D*} \tag{4.22}$$

$$W^* = W^{ND*} = CS^* + \pi^* \tag{4.23}$$

$$W^{D*} = CS^{D*} + \pi^{D*} (4.24)$$

From (4.22) we see that the new player after divestiture is a sibling of the low-cost firm. We also see this from total industry R&D expenditure:

$$I^* = I^{ND*} = (x_L^*)^2 + (x_H^*)^2 (4.25)$$

$$I^{D*} = 2(x_L^*)^2 + (x_H^*)^2 (4.26)$$

**Proposition 12** Comparing the divestiture scenario with the no divestiture scenario, for any  $\alpha \in ]0;1]$ , (i) industry profits are lower  $(\pi^{D*} < \pi^*)$ ; (ii) consumer surplus is higher  $(CS^{D*} > CS^*)$ ; (iii) there exists some nonempty interval  $Z^W \equiv [\underline{c_L}; c_L^W]$  such that welfare is higher  $(W^{D*} > W^*)$  if and only if  $c_L \in Z^W$ .

As to be expected in a Cournot setting, industry profits decrease as the number of firms increase in the divestiture scenario. In contrast, consumers are better off after a divestiture. Output increases which corresponds to lowered prices to be paid by consumers. More inter-

estingly, total welfare increases with a divestiture measure if the difference in marginal cost is sufficiently large. This is caused by a strong competition effect.

**Proposition 13** For any  $\alpha \in [0;1]$  without divestiture, (i) there exists some nonempty interval  $Z^{\pi} \equiv [c_L^{\pi}; c_H[$  such that industry profits  $(\pi^*)$  are increasing in  $\alpha$  if and only if  $c_L \in Z^{\pi}$ ; (ii) consumer surplus  $(CS^*)$  is decreasing in  $\alpha$ ; (iii) welfare  $(W^*)$  is decreasing in  $\alpha$ .

Ex ante, i.e. without any divestiture actually taking place, industry profits are only increasing if the difference in marginal cost is sufficiently small. The positive effect on firm H's profit outweighs a potential negative effect on firm L's profit then. The more likely a divestiture is the lower the overall output resulting in lower consumer surplus. The negative effect on consumer surplus dominates a potential positive profit effect. Therefore, ex ante welfare is decreasing in  $\alpha$ .

**Proposition 14** For any  $\alpha \in ]0;1]$  under the divestiture scenario, (i) industry profits  $(\pi^{D*})$  are increasing in  $\alpha$ ; (ii) consumer surplus  $(CS^{D*})$  is decreasing in  $\alpha$ ; (iii) there exists some nonempty interval  $Z^{W^D} \equiv \left[\underline{c_L}; c_L^{W^D}\right]$  such that welfare  $(W^{D*})$  is decreasing in  $\alpha$  if and only if  $c_L \in Z^{W^D}$ .

Proposition 14 states how an increase in  $\alpha$  affects the ex post divestiture scenario. Ex post industry profits are the higher the more likely a divestiture has been, while consumers suffer from a threat of divestiture. This is also true for welfare if there is a sufficiently large difference in marginal cost.

Finally, we look at the impact of an increase in  $\alpha$  on the industry-wide rate of innovation measured by total industry investments.

**Proposition 15** For any  $\alpha \in [0, 1]$ , (i)  $I^*$  is decreasing in  $\alpha$ ; (ii)  $I^{D*}$  is decreasing in  $\alpha$ .

A threat of divestiture does decrease ex ante innovation. A higher probability of divestiture also decreases investments in the divestiture scenario.<sup>2</sup>

We now want to present some potential implications for competition policy. Answering the question whether a divestiture was the right decision from a welfare maximizing point of

As we consider investments as recurring fixed cost, investments are generally higher after divestment:  $I^D = I + x_L^2$ .

view is of particular interest. Consumers are better off, while industry profits decrease after divestment. A divestiture is only welfare improving if the difference in marginal cost between high- and low-cost firm was sufficiently large. Thus, a potential regulator should only force a divestment if the dominance of one firm is sufficiently high. Therefore, intensified competition is not generally welfare improving. Considering ex ante market outcomes, welfare is the lower the more likely a divestiture is. Additionally, a divestiture threat has a negative impact on the industry-wide rate of innovation. A compensation scheme for the low-cost firm might keep it from restricting it innovative investments if there is a divestiture threat. This could lower the negative impact of a divestiture threat on ex ante welfare. In essence, a divestiture may be welfare enhancing if the market dominance of one firm is relatively strong. It should be carried out quickly as it has a negative welfare impact ex ante any divestiture. This negative impact might be lowered if there is a compensation scheme for the low-cost firm.

#### 4.4 Conclusion

This paper has analyzed strategic R&D investments in a Cournot model wherein one efficient low-cost firm competes against one less efficient high-cost firm, assuming that the low-cost firm is facing a threat of potential divestiture for which the firms account. In our 2-stage model, both firms can not only choose output, but also decrease their marginal cost via process innovations (stage 1). Even without innovation, firms are heterogenous, as they have different constant marginal cost. Between stages 1 and 2, there might be a divestiture of the low-cost firm. In case of divestiture firms face a third low-cost competitor and again compete in Cournot fashion. We assume the stage 1 R&D investments to be irreversible. Thus, we do not allow for any changes in investment in stage 2. The key results derived are the following:

An actual divestiture harms both high- and low-cost firm. A threat of divestiture reduces market power of the low-cost firm as it reacts by lowering its R&D investment. If the low-cost firm is very dominant a threat of divestiture may increase its profit. Considering the high-cost firm, a threat of divestiture stimulates its R&D investment if there is a weak dominance of the low-cost firm only. Additionally, the divestiture threat results in increasing profit of the high-cost firm ex ante any divestiture.

A divestiture only improves welfare if the low-cost firm sufficiently dominates the high-cost

firm. Accordingly, a potential regulator should only force a divestment of the low-cost firm if it highly dominates the high-cost firm. It should be carried out quickly as it has a negative welfare impact ex ante any divestiture. This negative impact might be lowered if there is a compensation scheme for the low-cost firm. The industry-wide rate of innovation is lowered if a divestiture gets more likely.

Our model offers several avenues for extension. First, our results should also hold if ex ante asymmetry focuses on the efficiency of cost reduction instead of on different marginal cost. Second, a deeper investigation how a compensation scheme for the low-cost firm influences its strategic behavior and welfare would be interesting. Furthermore, we restrict our attention to a duopoly case ex ante. A generalized oligopoly model should not contrast our implications. As the distribution of productivity matters, we expect different results if the new player adopts the high-cost firm cost structure. Finally, an empirical test of our model by analyzing industries where a regulator forced a horizontal divestiture promises exciting insights.

## 4.A Appendix

**Proof of Proposition 8.** (i) From (4.9) we have

$$\frac{dx_L^*}{d\alpha} = -6\frac{\Upsilon}{(-7\alpha^2 + 26\alpha + 224)^2} \tag{4.27}$$

where  $\Upsilon \equiv 3968c_H - 5888c_L - 672\alpha + 21\alpha^2 + 1120\alpha c_L - 448\alpha c_H - 119\alpha^2 c_L + 98\alpha^2 c_H + 1920$ .  $\frac{dx_L^*}{d\alpha} < 0$  if and only if  $\Upsilon > 0$ . As  $\Upsilon$  is decreasing in  $\alpha$  it suffices to show  $\Upsilon > 0$  for  $\alpha = 0$ .

$$\Upsilon|_{\alpha=0} = 3968c_H - 5888c_L + 1920 \tag{4.28}$$

As  $c_H > c_L$ ,  $\Upsilon > 0$  and  $\frac{dx_L^*}{d\alpha} < 0$  for all  $\alpha \in [0; 1]$ .

(ii) From (4.10) we have

$$\frac{dx_H^*}{d\alpha} = 12 \frac{\Phi}{(-7\alpha^2 + 26\alpha + 224)^2} \tag{4.29}$$

where  $\Phi \equiv 336\alpha - 2096c_H + 2048c_L + 21\alpha^2 - 560\alpha c_H + 224\alpha c_L - 98\alpha^2 c_H + 77\alpha^2 c_L + 48$ .  $\frac{dx_H^*}{d\alpha} > 0$  if and only if  $\Phi > 0$ .  $\Phi$  is positive if and only if

$$c_L > \frac{2096c_H - 336\alpha - 21\alpha^2 + 560\alpha c_H + 98\alpha^2 c_H - 48}{224\alpha + 77\alpha^2 + 2048} \equiv c_L^{x_H}$$

$$(4.30)$$

 $Z^{x_H} \equiv \left[c_L^{x_H}; c_H\right]$  is nonempty if  $c_H - c_L^{x_H} > 0$ .

$$c_H - c_L^{x_H} = 3\left(112\alpha + 7\alpha^2 + 16\right) \frac{1 - c_H}{224\alpha + 77\alpha^2 + 2048} > 0$$
 (4.31)

for any  $\alpha \in [0; 1]$ . Thus,  $\frac{dx_H^*}{d\alpha} > 0$  if and only if  $c_L \in Z^{x_H}$ .

**Proof of Proposition 9.** (i) From (4.11) we have

$$\frac{dq_L^*}{d\alpha} = 24 \frac{\Omega}{(-26\alpha + 7\alpha^2 - 224)^2}$$
 (4.32)

where  $\Omega \equiv 56\alpha - 312c_H + 640c_L - 7\alpha^2 + 168\alpha c_H - 224\alpha c_L + 7\alpha^2 c_L - 328$ .  $\frac{dq_L^*}{d\alpha} < 0$  if and only if  $\Omega < 0$ . As  $\Omega$  is increasing in  $\alpha$  it suffices to show  $\Omega < 0$  at  $\alpha = 1$ .

$$\Omega|_{\alpha=1} = -9\left(16c_H - 47c_L + 31\right) \tag{4.33}$$

As  $c_H > c_L$ ,  $\Omega < 0$  and  $\frac{dq_L^*}{d\alpha} < 0$  for all  $\alpha \in [0;1]$ . This is also true for  $q_L^{D*}$  as  $q_L^{D*} = \frac{3}{4}q_L^*$ .

(ii) From (4.15) we have

$$\frac{dq_H^*}{d\alpha} = 6 \frac{\Xi}{(-26\alpha + 7\alpha^2 - 224)^2} \tag{4.34}$$

where  $\Xi \equiv 224\alpha - 1472c_H + 768c_L + 35\alpha^2 - 896\alpha c_H + 672\alpha c_L - 98\alpha^2 c_H + 63\alpha^2 c_L + 704$ .  $\frac{dq_H^*}{d\alpha} > 0$  if and only if  $\Xi > 0$ .  $\Xi$  is positive if and only if

$$c_L > \frac{1472c_H - 224\alpha - 35\alpha^2 + 896\alpha c_H + 98\alpha^2 c_H - 704}{672\alpha + 63\alpha^2 + 768} \equiv c_L^{q_H}$$

$$(4.35)$$

As  $\underline{c_L} > c_L^{q_H}$  is always satisfied for  $\alpha \in [0;1]$ ,  $\frac{dq_H^*}{d\alpha} > 0$  is always true.  $q_H^{D*} = \frac{3}{4}q_H - \eta$  with  $\frac{d\eta}{d\alpha} < 0$ . Thus,  $\frac{dq_D^{D*}}{d\alpha} > 0$  is true for any  $\alpha \in [0;1]$ .

**Proof of Proposition 10.** From (4.13) we have

$$\frac{d\pi_L^*}{d\alpha} = \frac{\Delta\Gamma}{\left(-26\alpha + 7\alpha^2 - 224\right)^3} \tag{4.36}$$

where

 $\Gamma \equiv 33\,536c_H - 18\,816\alpha - 32\,768c_L + 4368\alpha^2 - 147\alpha^3 - 18\,816\alpha c_H + 37\,632\alpha c_L + 4704\alpha^2 c_H - 686\alpha^3 c_H - 9072\alpha^2 c_L + 833\alpha^3 c_L - 768$ 

$$\Delta \equiv -12(4(1 - (4c_L - 3c_H)) - \alpha(1 - c_L)).$$

 $(-26\alpha + 7\alpha^2 - 224)^3 < 0$ ,  $\Delta$  is strictly increasing in  $\alpha$ . Thus, it suffices to show  $\Delta < 0$  at  $\alpha = 1$ :

$$\Delta|_{\alpha=1} = 36 \left( 5c_L - 4c_H - 1 \right) < 0 \tag{4.37}$$

Hence,  $\frac{d\pi_L^*}{d\alpha} > 0$  if and only if  $\Gamma > 0$ .  $\Gamma$  is positive if and only if

$$c_{L} < c_{L}^{\pi_{L}}$$

$$\equiv (18816\alpha - 33536c_{H} - 4368\alpha^{2} + 147\alpha^{3} + 18816\alpha c_{H} - 4704\alpha^{2}c_{H} + 686\alpha^{3}c_{H} + 768)$$

$$\frac{1}{37632\alpha - 9072\alpha^{2} + 833\alpha^{3} - 32768}$$

$$(4.38)$$

 $Z^{\pi_L} \equiv \left[\underline{c_L}; c_L^{\pi_L}\right]$  is nonempty if  $c_L^{\pi_L} - \underline{c_L} > 0.$ 

$$c_L^{\pi_L} - \underline{c_L} = -\frac{1}{12} (c_H - 1) \left( -26\alpha + 7\alpha^2 - 224 \right)$$

$$\frac{-26880\alpha - 2016\alpha^2 + 833\alpha^3 + 8704}{(11\alpha + 16) (37632\alpha - 9072\alpha^2 + 833\alpha^3 - 32768)}$$
(4.39)

As  $\frac{(1-c_H)(-26\alpha+7\alpha^2-224)}{12(11\alpha+16)(37632\alpha-9072\alpha^2+833\alpha^3-32768)} > 0,$  $c_L^{\pi_L} - \underline{c_L} > 0 \text{ requires } -26880\alpha - 2016\alpha^2 + 833\alpha^3 + 8704 > 0.$ 

This is true for

$$\alpha < 0.32. \tag{4.40}$$

From (4.17) we have

$$\frac{d\pi_H^*}{d\alpha} = \frac{24\Psi}{\left(-7\alpha^2 + 26\alpha + 224\right)^3} \tag{4.41}$$

where

$$\begin{split} \Psi &\equiv -686\alpha^4 c_H^2 + 539\alpha^4 c_H c_L + 833\alpha^4 c_H - 539\alpha^4 c_L - 147\alpha^4 - 13720\alpha^3 c_H^2 + 20146\alpha^3 c_H c_L + \\ 7294\alpha^3 c_H &- 8463\alpha^3 c_L^2 - 3220\alpha^3 c_L - 2037\alpha^3 - 62832\alpha^2 c_H^2 + 79968\alpha^2 c_H c_L + 45696\alpha^2 c_H - \\ 17136\alpha^2 c_L^2 - 45696\alpha^2 c_L - 207136\alpha c_H^2 + 402304\alpha c_H c_L + 11968\alpha c_H - 195840\alpha c_L^2 - 10624\alpha c_L - \\ 672\alpha - 253952c_H^2 + 567296c_H c_L - 59392c_H - 282624c_L^2 - 2048c_L + 30720. \end{split}$$

 $\frac{d\pi_H^*}{d\alpha} > 0$  if and only if  $\Psi > 0$ .  $\Psi$  is positive if and only if

$$c_{L} > c_{L}^{\pi_{H}}$$

$$\equiv -\frac{1}{6(2821\alpha^{3} + 5712\alpha^{2} + 65280\alpha + 94208)}$$

$$((-26\alpha + 7\alpha^{2} - 224)(c_{H} - 1)$$

$$\sqrt{304640\alpha + 672\alpha^{2} + 13328\alpha^{3} + 5929\alpha^{4} + 692224}$$

$$+10624\alpha - 567296c_{H} + 45696\alpha^{2} + 3220\alpha^{3} + 539\alpha^{4}$$

$$-402304\alpha c_{H} - 79968\alpha^{2}c_{H} - 20146\alpha^{3}c_{H} - 539\alpha^{4}c_{H} + 2048)$$

$$(4.42)$$

As  $\underline{c_L} > c_L^{\pi_H}$  is always satisfied,  $\frac{d\pi_H^*}{d\alpha} > 0$  is always true.

**Proof of Proposition 11.** (i) From (4.14) we have

$$\frac{d\pi_L^{D*}}{d\alpha} = \frac{\Delta\xi}{\left(26\alpha - 7\alpha^2 + 224\right)^3} \tag{4.43}$$

 $-47\,040\alpha c_L - 4704\alpha^2 c_H + 686\alpha^3 c_H + 9366\alpha^2 c_L - 833\alpha^3 c_L - 13\,008$  where  $\xi \equiv 21\,168\alpha - 46\,640c_H + 59\,648c_L - 4662\alpha^2 + 147\alpha^3 + 25\,872\alpha c_H$   $-47\,040\alpha c_L - 4704\alpha^2 c_H + 686\alpha^3 c_H + 9366\alpha^2 c_L - 833\alpha^3 c_L - 13\,008,$   $\Delta \equiv -12(4(1 - (4c_L - 3c_H)) - \alpha(1 - c_L)).$   $\left(26\alpha - 7\alpha^2 + 224\right)^3 > 0, \ \Delta < 0.^3 \ \frac{d\pi_L^{D*}}{d\alpha} > 0 \text{ if and only if } \xi < 0. \text{ As } \xi \text{ is strictly increasing in } \alpha \text{ it suffices to show } \xi < 0 \text{ for } \alpha = 1.$ 

$$\xi|_{\alpha=1} = 21\,141c_L - 24\,786c_H + 3645\tag{4.44}$$

As  $c_H > c_L$ ,  $\xi < 0$  and  $\frac{d\pi_L^{D*}}{d\alpha} > 0$  for all  $\alpha \in [0; 1]$ .

(ii) From (4.18) we have

$$\frac{d\pi_H^{D*}}{d\alpha} = \frac{24\theta}{(26\alpha - 7\alpha^2 + 224)^3} \tag{4.45}$$

where

$$\theta = 539\alpha^4 c_H c_L - 686\alpha^4 c_H^2 + 833\alpha^4 c_H - 539\alpha^4 c_L - 147\alpha^4 - 13720\alpha^3 c_H^2 + 20734\alpha^3 c_H c_L +$$

<sup>&</sup>lt;sup>3</sup>See proof of Proposition 4.

 $\begin{aligned} &6706\alpha^3c_H - 8820\alpha^3c_L^2 - 3094\alpha^3c_L - 1806\alpha^3 - 52\,248\alpha^2c_H^2 + 63\,840\alpha^2c_Hc_L + 40\,656\alpha^2c_H - \\ &12\,096\alpha^2c_L^2 - 39\,648\alpha^2c_L - 504\alpha^2 - 149\,680\alpha c_H^2 + 290\,560\alpha c_Hc_L + 8800\alpha c_H - 138\,240\alpha c_L^2 - \\ &14\,080\alpha c_L + 2640\alpha - 272\,096c_H^2 + 674\,816c_Hc_L - 130\,624c_H - 368\,640c_L^2 + 62\,464c_L + 34\,080. \\ &\frac{d\pi_H^{D*}}{d\alpha} < 0 \text{ if and only if } \theta < 0. \ \theta \text{ is negative if and only if} \end{aligned}$ 

$$c_{L} < c_{L}^{\pi_{H}^{D}}$$

$$\equiv \frac{1}{72(245\alpha^{3} + 336\alpha^{2} + 3840\alpha + 10240)}$$

$$((c_{H} - 1)(26\alpha - 7\alpha^{2} + 224))$$

$$\sqrt{167552\alpha - 33600\alpha^{2} + 6272\alpha^{3} + 5929\alpha^{4} + 1079296}$$

$$-14080\alpha + 674816c_{H} - 39648\alpha^{2} - 3094\alpha^{3} - 539\alpha^{4}$$

$$+290560\alpha c_{H} + 63840\alpha^{2}c_{H} + 20734\alpha^{3}c_{H} + 539\alpha^{4}c_{H} + 62464)$$

$$(4.46)$$

 $Z^{\pi_H^D} \equiv \left[\underline{c_L}; c_L^{\pi_H^D}\right]$  is nonempty if  $c_L^{\pi_H^D} - \underline{c_L} > 0$ .

$$c_L^{\pi_H^D} - c_L > 0 (4.47)$$

for any  $\alpha \in [0; 1]$ . Thus,  $\frac{d\pi_H^{D*}}{d\alpha} < 0$  if and only if  $c_L \in Z^{\pi_H^D}$ .

**Proof of Proposition 12.** (i) For  $\pi^{D*} < \pi^*$  we have

$$\pi^{D*} = -(7\alpha + 2)(7\alpha - 34) \frac{(-\alpha - 16c_L + 12c_H + \alpha c_L + 4)^2}{(-26\alpha + 7\alpha^2 - 224)^2} + 2((26\alpha + 192c_L - 200c_H - 7\alpha^2 - 84\alpha c_L + 58\alpha c_H + 7\alpha^2 c_H + 8) \frac{46\alpha + 576c_L - 712c_H + 7\alpha^2 + 180\alpha c_L - 226\alpha c_H - 7\alpha^2 c_H + 136}{(-26\alpha + 7\alpha^2 - 224)^2})$$

$$(4.48)$$

and

$$\pi^* = -(7\alpha + 8) (7\alpha - 40) \frac{(-\alpha - 16c_L + 12c_H + \alpha c_L + 4)^2}{(-26\alpha + 7\alpha^2 - 224)^2} + (20\alpha + 96c_L - 128c_H - 7\alpha^2 - 78\alpha c_L + 58\alpha c_H + 7\alpha^2 c_H + 32) \frac{40\alpha + 480c_L - 640c_H + 7\alpha^2 + 186\alpha c_L - 226\alpha c_H - 7\alpha^2 c_H + 160}{(-26\alpha + 7\alpha^2 - 224)^2}$$

$$(4.49)$$

We need to show

$$\pi^{D*} - \pi^* = \frac{\varrho - \epsilon}{\left(-26\alpha + 7\alpha^2 - 224\right)^2} < 0 \tag{4.50}$$

where  $\epsilon = 252 \left(-\alpha + 12c_H - 16c_L + \alpha c_L + 4\right)^2$  and  $\varrho = 3328\alpha - 24832c_H + 30720c_L + 696\alpha^2 - 140\alpha^3 - 49\alpha^4 + 202880c_H^2 + 175104c_L^2 - 16692\alpha^2 c_H^2 - 1988\alpha^3 c_H^2 - 49\alpha^4 c_H^2 - 15732\alpha^2 c_L^2 - 27392\alpha c_H + 20736\alpha c_L - 380928c_H c_L + 16000\alpha c_H^2 + 264\alpha^2 c_H + 2128\alpha^3 c_H + 98\alpha^4 c_H - 8064\alpha c_L^2 - 1656\alpha^2 c_L - 1848\alpha^3 c_L + 33120\alpha^2 c_H c_L + 1848\alpha^3 c_H c_L - 4608\alpha c_H c_L - 2944$ 

 $\pi^{D*} - \pi^* < 0$  requires  $\varrho - \epsilon < 0$ . This is always true for  $c_L \in [\underline{c_L}; c_H]$ . Consequently,  $\pi^{D*} < \pi^*$  is always satisfied for  $\alpha \in ]0;1]$ .

(ii) For  $CS^{D*} > CS^*$  we need to show

$$CS^{D*} - CS^* = \frac{\varsigma \check{z}}{(-26\alpha + 7\alpha^2 - 224)^2} > 0$$
 (4.51)

where  $\varsigma = -\alpha + 20c_H + 48c_L + 28\alpha c_H - 27\alpha c_L - 68$ ,

$$\check{z} = -18 \left( \alpha (c_L - 1) + 12c_H - 16c_L + 4 \right)$$

As  $\varsigma < 0$  and  $\check{z} < 0$  for  $\alpha \in [0; 1]$ ,  $CS^{D*} > CS^*$  is always true.

(iii) For  $W^{D*} > W^*$  we need to show

$$W^{D*} - W^* = \frac{\varsigma \dot{z} + \varrho - \epsilon}{(-26\alpha + 7\alpha^2 - 224)^2} > 0 \tag{4.52}$$

 $W^{D*} - W^* > 0$  requires  $\zeta \check{z} + \varrho > \epsilon$ . This is true if

$$c_{L} < c_{L}^{W}$$

$$\equiv \frac{1}{8640\alpha + 15498\alpha^{2} - 124416} (\sqrt{2}\sqrt{-448\alpha + 107\alpha^{2} + 728})$$

$$(-672 + 672c_{H} + 21\alpha^{2} - 21\alpha^{2}c_{H} + 78\alpha c_{H} - 78\alpha)$$

$$+7200\alpha - 144384c_{H} - 810\alpha^{2} - 924\alpha^{3}$$

$$+1440\alpha c_{H} + 16308\alpha^{2}c_{H} + 924\alpha^{3}c_{H} + 19968)$$

$$(4.53)$$

 $Z^W \equiv [\underline{c_L}; c_L^W]$  is nonempty if  $c_L^W - \underline{c_L} > 0$ .

$$c_L^W - \underline{c_L} > 0 \tag{4.54}$$

for any  $\alpha \in ]0;1]$ . Thus,  $W^{D*} > W^*$  if and only if  $c_L \in Z^W$ .

**Proof of Proposition 13.** (i) From (4.49) we get

$$\frac{d\pi^*}{d\alpha} = \frac{12\vartheta}{(-26\alpha + 7\alpha^2 - 224)^3} \tag{4.55}$$

where  $\vartheta \equiv 75\,840\alpha - 6144c_H + 122\,880c_L - 36\,288\alpha^2 + 9030\alpha^3 + 147\alpha^4 + 105\,472c_H^2 + 40\,960c_L^2 + 69\,216\alpha^2c_H^2 + 35\,672\alpha^3c_H^2 + 1372\alpha^4c_H^2 - 148\,512\alpha^2c_L^2 + 39\,326\alpha^3c_L^2 - 833\alpha^4c_L^2 + 310\,656\alpha c_H - 462\,336\alpha c_L - 204\,800c_Hc_L + 640\,064\alpha c_H^2 - 181\,440\alpha^2c_H - 5376\alpha^3c_H - 2352\alpha^4c_H + 1026\,560\alpha c_L^2 + 254\,016\alpha^2c_L - 12\,684\alpha^3c_L + 2058\alpha^4c_L + 43\,008\alpha^2c_Hc_L - 65\,968\alpha^3c_Hc_L - 392\alpha^4c_Hc_L - 1590\,784\alpha c_Hc_L - 58\,368.$ 

 $\frac{d\pi^*}{d\alpha} > 0$  requires  $\vartheta < 0$ .  $\vartheta$  is negative if and only if

$$c_L \in [c_L^{\pi}; c_H] \tag{4.56}$$

where

$$c_L^{\pi} \equiv \frac{1}{1026560\alpha - 148512\alpha^2 + 39326\alpha^3 - 833\alpha^4 + 40960} (\sqrt{3}\sqrt{44800\alpha - 36456\alpha^2 - 4312\alpha^3 + 2009\alpha^4 + 10240}$$

$$(-448 - 52\alpha + 448c_H + 14\alpha^2 - 14\alpha^2c_H + 52\alpha c_H) + 196\alpha^4c_H$$

$$+231168\alpha + 102400c_H - 127008\alpha^2 + 6342\alpha^3 - 1029\alpha^4$$

$$+795392\alpha c_H - 21504\alpha^2c_H + 32984\alpha^3c_H - 61440)$$

$$(4.57)$$

 $Z^{\pi} \equiv [c_L^{\pi}; c_H]$  is nonempty if  $c_H - c_L^{\pi} > 0$ .

$$c_H - c_L^{\pi} > 0 \tag{4.58}$$

for any  $\alpha \in [0; 1]$ . Thus,  $\vartheta < 0$  and  $\frac{d\pi^*}{d\alpha} > 0$  if and only if  $c_L \in Z^{\pi}$ .

(ii) From (4.19) we get 
$$\frac{dCS^*}{d\alpha} = 36 \frac{\check{r}g}{(26\alpha - 7\alpha^2 + 224)^3}$$
 (4.59)

where  $\check{r} \equiv (16(c_H + c_L) - \alpha(1 + 13c_L - 14c_H) - 32),$   $g \equiv -448\alpha + 2720c_H - 3328c_L - 7\alpha^2 + 224\alpha c_H + 224\alpha c_L + 98\alpha^2 c_H - 91\alpha^2 c_L + 608.$ As  $\check{r} < 0$  for  $\alpha \in [0; 1], \frac{dCS^*}{d\alpha} < 0$  if and only if g > 0. As g is strictly decreasing in  $\alpha$  for  $\alpha \in [0; 1]$  it suffices to show g > 0 for  $\alpha = 1$ .

$$g|_{\alpha=1} = 9(338c_H - 355c_L + 17) \tag{4.60}$$

As  $c_H > c_L$ , g > 0 and  $\frac{dCS^*}{d\alpha} < 0$  for all  $\alpha \in [0; 1]$ .

(iii) 
$$\frac{dW^*}{d\alpha} = \frac{-12\acute{Z}}{(-26\alpha + 7\alpha^2 - 224)^3}$$
 (4.61)

where  $\acute{Z} \equiv -1372\alpha^4c_H^2 + 392\alpha^4c_Hc_L + 2352\alpha^4c_H + 833\alpha^4c_L^2 - 2058\alpha^4c_L - 147\alpha^4 - 31556\alpha^3$   $c_H^2 + 58324\alpha^3c_Hc_L + 4788\alpha^3c_H - 35777\alpha^3c_L^2 + 13230\alpha^3c_L - 9009\alpha^3 - 55104\alpha^2c_H^2 - 42000\alpha^2c_Hc_L + 152208\alpha^2c_H + 135408\alpha^2c_L^2 - 228816\alpha^2c_L + 38304\alpha^2 - 515072\alpha c_H^2 + 1366432\alpha c_Hc_L - 336288\alpha c_H - 886016\alpha c_L^2 + 405600\alpha c_L - 34656\alpha + 25088c_H^2 + 175616c_Hc_L - 225792c_H - 200704c_L^2 + 225792c_L$   $\frac{dW^*}{d\alpha} < 0$ , if  $\acute{Z} < 0$ .  $\acute{Z}$  is negative if and only if

$$c_L < c_L^W \tag{4.62}$$

where

$$c_{L} < c_{L}^{W}$$

$$\equiv \frac{1}{886016\alpha - 135408\alpha^{2} + 35777\alpha^{3} - 833\alpha^{4} + 200704}$$

$$(\sqrt{21}\sqrt{8512\alpha - 3660\alpha^{2} - 630\alpha^{3} + 287\alpha^{4} + 3024}$$

$$(-52\alpha - 448 + 448c_{H} + 14\alpha^{2} + 52\alpha c_{H} - 14\alpha^{2}c_{H})$$

$$+202800\alpha + 87808c_{H} - 114408\alpha^{2} + 6615\alpha^{3} + 683216\alpha c_{H}$$

$$-1029\alpha^{4} - 21000\alpha^{2}c_{H} + 29162\alpha^{3}c_{H} + 196\alpha^{4}c_{H} + 112896)$$

$$(4.63)$$

As  $c_H < c_L^W$  is always satisfied for  $\alpha \in [0; 1], \frac{dW^*}{d\alpha} < 0$  is always true.

**Proof of Proposition 14.** (i) From (4.48) we get

$$\frac{d\pi^{D*}}{d\alpha} = -\frac{12\chi}{(-26\alpha + 7\alpha^2 - 224)^3} \tag{4.64}$$

where  $\chi = -2744\alpha^4 c_H^2 + 1470\alpha^4 c_H c_L + 4018\alpha^4 c_H + 833\alpha^4 c_L^2 - 3136\alpha^4 c_L - 441\alpha^4 - 63112\alpha^3$  $c_H^2 + 108612\alpha^3 c_H c_L + 17612\alpha^3 c_H - 57974\alpha^3 c_L^2 + 7336\alpha^3 c_L - 12474\alpha^3 - 152544\alpha^2 c_H^2 + 41832\alpha^2 c_H c_L + 263256\alpha^2 c_H + 148512\alpha^2 c_L^2 - 338856\alpha^2 c_L + 37800\alpha^2 - 909184\alpha c_H^2 + 2187312\alpha c_H c_L - 368944\alpha c_H - 1365248\alpha c_L^2 + 543184\alpha c_L - 87120\alpha - 528704c_H^2 + 1237248c_H c_L - 179840c_H - 520192c_L^2 - 196864c_L + 188352$ 

 $\frac{d\pi^{D*}}{d\alpha} > 0$  requires  $\chi > 0$ .  $\chi$  is positive for all  $\alpha \in ]0;1]$  and  $c_L \in [\underline{c_L}; c_H[$ .

(ii) From (4.20) we get

$$\frac{dCS^{D*}}{d\alpha} = 144 \frac{j\varpi}{(26\alpha - 7\alpha^2 + 224)^3}$$
 (4.65)

where  $j \equiv -252\alpha + 1516c_H - 1984c_L + 28\alpha c_H + 224\alpha c_L + 49\alpha^2 c_H - 49\alpha^2 c_L + 468$ ,  $\varpi \equiv 2c_H + 16c_L + 7\alpha(c_H - c_L) - 18$ .  $\varpi < 0$  for any  $\alpha \in ]0;1]$ , thus  $\frac{dCS^{D*}}{d\alpha} < 0$  if and only if j > 0. As j is strictly decreasing in  $\alpha$  for  $\alpha \in [0;1]$  it suffices to show j > 0 for  $\alpha = 1$ .

$$j|_{\alpha=1} = 27(59c_H - 67c_L + 8) \tag{4.66}$$

As  $c_H > c_L$ , j > 0 and  $\frac{dCS^{D*}}{d\alpha} < 0$  for all  $\alpha \in ]0;1]$ .

(iii) From (4.24) we get

$$\frac{dW^{D*}}{d\alpha} = -12 \frac{\nu}{(-26\alpha + 7\alpha^2 - 224)^3} \tag{4.67}$$

where  $\nu = -2744\alpha^4 c_H^2 + 1470\alpha^4 c_H c_L + 4018\alpha^4 c_H + 833\alpha^4 c_L^2 - 3136\alpha^4 c_L - 441\alpha^4 - 58\,996\alpha^3 c_H^2 + 100\,380\alpha^3 c_H c_L + 17\,612\alpha^3 c_H - 53\,858\alpha^3 c_L^2 + 7336\alpha^3 c_L - 12\,474\alpha^3 - 149\,016\alpha^2 c_H^2 + 66\,528\alpha^2 c_H c_L + 231\,504\alpha^2 c_H + 120\,288\alpha^2 c_L^2 - 307\,104\alpha^2 c_L + 37\,800\alpha^2 - 781\,168\alpha c_H^2 + 1904\,064\alpha c_H c_L - 341\,728\alpha c_H - 1155\,584\alpha c_L^2 + 407\,104\alpha c_L - 32\,688\alpha - 492\,320c_H^2 + 1480\,704c_H c_L - 496\,064c_H - 901\,120c_L^2 + 120\,68\alpha^2 c_H^2 + 120\,66\alpha^2 c_H^2 + 120\,6\alpha^2 c_H^2 + 120\,6\alpha^2 c_H^2 + 120\,6\alpha^2 c_H^2 + 120\,6\alpha^2 c_H^2 + 120\,6\alpha^2$ 

 $321\,536c_L + 87\,264$ 

 $\frac{dW^{D*}}{d\alpha}$  < 0 requires v < 0. v is negative if and only if

$$c_{L} < c_{L}^{W^{D}}$$

$$\equiv \frac{1}{1155584\alpha - 120288\alpha^{2} + 53858\alpha^{3} - 833\alpha^{4} + 901120}$$

$$(\sqrt{7}\sqrt{320528\alpha - 122880\alpha^{2} - 11284\alpha^{3} + 8239\alpha^{4} + 297472}$$

$$(-224 - 26\alpha + 224c_{H} + 7\alpha^{2} - 7\alpha^{2}c_{H} + 26\alpha c_{H})$$

$$+203552\alpha + 740352c_{H} - 153552\alpha^{2} + 3668\alpha^{3} - 1568\alpha^{4}$$

$$+952032\alpha c_{H} + 33264\alpha^{2}c_{H} + 50190\alpha^{3}c_{H} + 735\alpha^{4}c_{H} + 160768$$
(4.68)

 $Z^{W^D} \equiv \left[\underline{c_L}; c_L^{W^D}\right]$  is nonempty if  $c_L^{W^D} - \underline{c_L} > 0$ 

$$c_L^{W^D} - c_L > 0 (4.69)$$

for any  $\alpha \in ]0;1]$ . Thus,  $\frac{dW^{D*}}{d\alpha} < 0$  if and only if  $c_L \in Z^{W^D}$ .

#### Proof of Proposition 15. (i)

$$I^* = x_L^2 + x_H^2 = (-26\alpha + 7\alpha^2 - 224)^{-2}$$

$$((44\alpha - 192c_H + 256c_L - 7\alpha^2 - 128\alpha c_L + 84\alpha c_H + 7\alpha^2 c_L - 64)^2$$

$$+(-10\alpha + 256c_H - 192c_L - 7\alpha^2 - 128\alpha c_L$$

$$+84\alpha c_H + 58\alpha c_H - 4\alpha c_L + 7\alpha^2 c_H - 64)^2)$$

$$(4.70)$$

$$\frac{dI^*}{d\alpha} = -12 \frac{\iota}{(-26\alpha + 7\alpha^2 - 224)^3} \tag{4.71}$$

where  $\iota \equiv 171456\alpha - 915456c_H + 1148928c_L - 34272\alpha^2 + 10752\alpha^3 + 147\alpha^4 + 311296c_H^2 - 720896c_L^2 + 182112\alpha^2c_H^2 + 43904\alpha^3c_H^2 + 1372\alpha^4c_H^2 - 126336\alpha^2c_L^2 + 43400\alpha^3c_L^2 - 833\alpha^4c_L^2 + 194304\alpha c_H - 537216\alpha c_L + 292864c_H c_L + 1301312\alpha c_H^2 - 274176\alpha^2c_H - 11256\alpha^3c_H - 2352\alpha^4c_H + 1667072\alpha c_L^2 + 342720\alpha^2c_L - 10248\alpha^3c_L + 2058\alpha^4c_L - 90048\alpha^2c_H c_L - 76552\alpha^3c_H c_L - 392\alpha^4c_H c_L - 2796928\alpha c_H c_L - 116736$ 

 $\frac{dI^*}{d\alpha} < 0,$  if  $\iota < 0.$   $\iota$  is negative if and only if

$$c_{L} < c_{L}^{I}$$

$$\equiv -\frac{1}{-1667072\alpha + 126336\alpha^{2} - 43400\alpha^{3} + 833\alpha^{4} + 720896}$$

$$(\sqrt{3}\sqrt{-78848\alpha - 55104\alpha^{2} + 1372\alpha^{3} + 2009\alpha^{4} + 408320}$$

$$(448 + 52\alpha - 448c_{H} - 14\alpha^{2} + 14\alpha^{2}c_{H} - 52\alpha c_{H})$$

$$+268608\alpha - 146432c_{H} - 171360\alpha^{2} + 5124\alpha^{3} + 38276\alpha^{3}c_{H}$$

$$-1029\alpha^{4} + 1398464\alpha c_{H} + 45024\alpha^{2}c_{H} + 196\alpha^{4}c_{H} - 574464)$$

$$(4.72)$$

As  $c_H < c_L^I$  is always satisfied for  $\alpha \in [0;1], \, \frac{dI^*}{d\alpha} < 0$  is always true.

(ii)  $I^{D*} = I^* + (x_L^*)^2$  is decreasing in  $\alpha$  as  $I^*$  is decreasing in  $\alpha$  and  $x_L^*$  is decreasing in  $\alpha$ .

## Chapter 5

# Conclusion

The following chapter summarizes the main findings and discusses topics for further research.

Chapter 2 entitled Competition between Pay-TV and Public Service Broadcasting:

A Two-Sided Market Analysis investigates the behavior of two competing TV channels - Pay-TV and PSB assuming that consumers engage in mental accounting and develop a portfolio demand. While Pay-TV channels have to finance themselves by advertising and subscription revenue generated from viewers, public service broadcasters are financed by advertising income and public funds. It turns out that the Pay-TV channel decides to show no adverts altogether if viewers display a strong aversion to advertising. If the broadcasting fee is sufficiently high, the Pay-TV channel switches to a free-to-air channel and the PSB does not show any adverts at all. Taking fixed cost for Pay-TV into account, the Pay-TV channel exits the market if the broadcasting fee is sufficiently high. If the government maximizes the number of PSB viewers the broadcasting fee is set so that the Pay-TV channel leaves the market. There are several topics that are left for further research. A welfare analysis would be especially interesting to see how it is influenced by a market exit of the Pay-TV channel. Additionally, the integration of different program content could offer further insights.

Chapter 3 entitled Endogenous Merger Formation and Incentives to Invest in Cost Reducing Innovations analyzes endogenous horizontal merger formation in a market with three players where firms engage in process innovations. Firms differ in their investment efficiency for cost reducing innovations. The asymmetry in R&D efficiency significantly influences the merger decisions. For a low to moderate level of innovation cost the two most efficient firms

merge, while the least efficient firm remains independent. For a high level of innovation cost there is no merger whatsoever. Market outcomes without a merger are socially desirable. Welfare is either increased or decreased in the predicted merger case. Accounting for endogenous merger formation R&D subsidies are not necessarily positive depending on the R&D efficiency in the industry. R&D subsidies are only positive if they do not influence any merger decision. It would be of special interest to implement a payoff division rule to be applied to merged entities. Additionally, the possibility of new entrants could give new insights.

Chapter 4 entitled Horizontal Divestitures and R&D Incentives in Asymmetric Duopoly analyzes how a threat of horizontal divestiture affects R&D incentives and welfare in an asymmetric Cournot duopoly where an efficient low-cost firm competes against a less efficient high-cost firm. Firms account for a possible divestiture of the low-cost firm. We find that an actual divestiture measure harms both the high- and low-cost firm. A divestiture threat already reduces the low-cost firm's market power, as the firm reduces its R&D investment, and, thereby, its competitiveness. Nevertheless, the threat of divestiture may increase the low-cost firm's profit in case of strong market dominance. Considering the high-cost firm, a threat of divestiture stimulates its R&D investment if there is a low dominance of the low-cost firm only. A divestiture only improves welfare if the low-cost firm sufficiently dominates the high-cost firm. The industry-wide rate of innovation is lowered if a divestiture becomes more likely. The most interesting avenue for further research is the impact of a compensation scheme for the low-cost firm on strategic behavior and market outcomes.

Chapter 3 and 4 both account for process innovation. A different view on R&D could be obtained by focusing on product innovation rather than process innovation. All three models are restricted to two or three firms. A generalization to the oligopoly case would give additional insights. Additionally, empirical evidence could strengthen our results.

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